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Computer Vision viewing over Mathematics

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RESUME: The difference between having images of something and having a knowledge of the same thing has been a subject of reflection for centuries, and some of the most important Greek tragedies focused on that point. A great part of information now consists of images: image has become the language of many people. Authors have already written on the necessity of learning how to read images. In this article the interest of learning how to write images will be experimented in the context of Mathematics Teaching. Our experiments have taken place in the Electrical Engineering and Computer Science Department of the Institute of Technology of Valenciennes. Different kinds of students have been concerned aged from 15 until 25, in different circumstances: Festival of Science, Mathematics lectures or more specialized courses on Robotics and Vision. The experiments have been mainly used with low-cost material: free software like Python, OpenCV and cheap hardware like Raspberry Pi are sufficient to obtain pretty good results.

Mots clés : Computer Vision, Image, Mathematics, OpenCV, Python, Raspberry Pi.

1 EXPERIENCE CONTEXT

1.1 Hardware

Most experiments described in the article have been made on a Raspberry Pi 3. The Raspberry Pi Foundation is a UK based charity founded in 2009. This organization intends to help people all around the planet to access to digital world. To accomplish this goal, the Raspberry Pi Foundation has worked with academic, scientific and research community, and in particular with Cambridge University, for the development of a low cost and flexible hardware for experimenting with programming and electronics. This collaboration is at the origin of the Raspberry Pi family, and in particular the Raspberry Pi 3, which we have been working with.



fig 1: Raspberry Pi3

The Raspberry Pi is equipped with General Purpose Input Output pins which allow the use of a wide range of actuators and sensors. A huge number of low-cost projects based on Raspberry Pi has so been imagined by makers and gives a source of inspiration for students. Retro-gaming, drones designing,

1.2 Software

Python is an interpreted language created in 1991 by Guido van Rossum. Python interpreters are available for most operating systems, and in particular in available for the Rasbian Stretch version of Linux that is installed on Raspberry Pi. Python supports oriented-object programming and many libraries for Python use the object concept.

Free distributions of Python and of useful libraries for Python can be found on the internet. Among the libraries that are used for the work presented here are Numpy (for numerical work), Matplotlib (for graphical work), and OpenCV (for computer vision).

1.3 Students

The theater set of the experiences described here is a group of twelve students with no experience in computer vision. These students are given an initiation to computer vision in the context of Festival of Science, or Week of Mathematics, or High School Students Immersion at University. One goal of the initiation is to show them that Mathematics are a useful tool and are used in domains they appreciate like images construction.

2 PEDAGOGICAL METHOD

2.1 The DARE method

The experience is organized using the pedagogical method DARE developed by the authors.

DARE is the acronym for Discover, Apply and Resolve. This method can be perceived as a rewriting, for computer languages, of the classical three-steps tests developed for language teaching: first translating from foreign language to native language, then translating from native language to foreign language and last writing an essay in foreign language.

The Discover step is devoted to translating computer language (here python and OpenCV instruction) into

native language. Student are given scripts they have to run and comment. In the Apply step student have to answer very simple questions using the instructions seen in the first step: they simply translate basic works into computer language. The last step Resolve is the most complicated but also most interesting step. Students must use their judgement, creativity and skills to solve a specific problem, in link with the previous steps.

2.2 Writing a pedagogical sequence

A pedagogical sequence is generally written around a precise theme. When the theme is chosen, a specific problem in link with the theme is chosen. To avoid too much lost of generality, the “Object Tracking” problem will be chosen for this article as it can be linked to many themes.

First thing to do when writing the subject of the sequence is to begin by the end, that is “Resolve the problem”. A group of teachers solve the problem, compare their solutions and draw up a list of the different instructions used. These instructions correspond to the different steps of the resolution of the problem: these steps will give the subjects of the different exercises of the “Apply” part. Once again teachers have to propose a list of exercises for each subject, solve them and then write scripts corresponding to solutions of similar exercises. These solutions give the “Resolve” part.

In our example, we shall distinguish three main steps. When doing “Object Tracking” with a camera, the first mathematical object we encounter is the **color RGB image**.

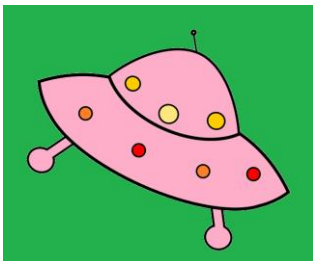


fig 2: Color Image

Usually, we have to convert this image in the HSV format. In both formats a color image can be thought as three **greyscale** (or intensity) images.

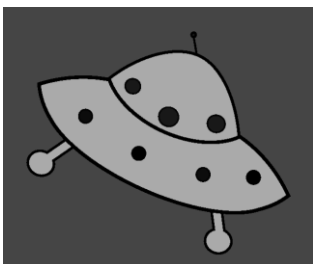


fig 3: Greyscale Image (Hue component)

These greyscale images are usually transformed in **black and white** images using a threshold method.

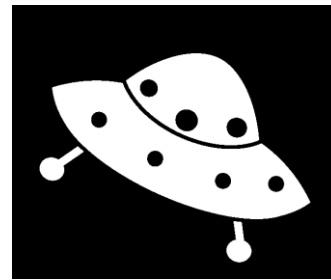


fig 4: Black and white image

We have then got three main steps for our pedagogical sequence: first Black and White images, then Greyscale images, and finally Color images. We can now begin our travel to the what students sometimes fell like as an alien world, that is the world of Mathematics, but viewed from the computer vision field.

3 BLACK AND WHITE

3.1 Set theory

One of the first application of working with Black and White images to play mathematics is Set Theory, and in particular Venn diagrams. Black and White images are indeed logical images. So, when the following (fig 5) Venn diagram is sketched:

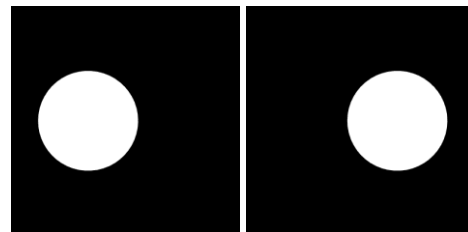


fig 5: set-image A and set-image B

the image is not only a representation of the set A: A is the set of white pixels. As opencv performs logical operations on image, thanks to the instruction `cv2.bitwise_or`, the union of two sets can at the same time be computed and illustrated: the computation is its own representation:

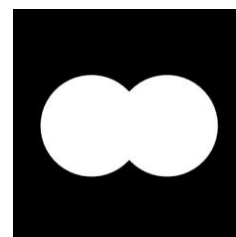


fig 6: set-image A OR B

3.2 Area calculation

This application of Black and White images is a very fruitful one. It can be used at different levels, such as:

- Calculating basic areas:

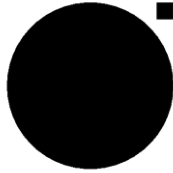


fig 7: What is the area of the disk?

- Integration:



fig 8: What is the value of $\ln 3$?

- Probabilities:

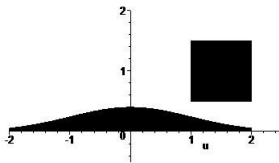


fig 9: What is the area under Gauss curve?

- Cinematics:



fig 10: What is the center of mass of the triangle

- Modeling:

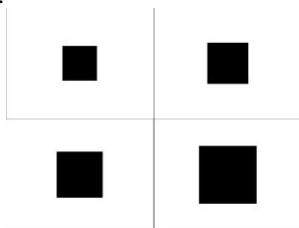


fig 11: Same object viewed from 32.5, 27.5, 24.5 and 19.5 cm. What is the model?

3.3 Geometric transformations

One more time, different levels can be performed, from the classical affine transformations to the more complicated projective transformations.

- Rotation:

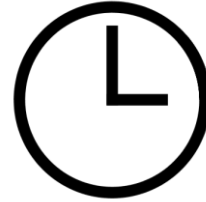


fig 10: Set the correct time: 11:45

- General affine transformation given by three points and their images
- General projective transformations given by four points and their images

4 GREYSCALE IMAGES

4.1 Statistics

4.2 Geometry: barycentric calculations

4.3 Calculus

4.4 Fourier analysis

5 COLOR IMAGES

5.1 Linear algebra

5.2 Optimisation

Bibliography

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