# On the Eco-driving Trajectory for Tramway System

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**Simon Enjalbert** *, **Yassine Boukal** *

* *Univ. Polytechnique Hauts-de-France, CNRS, UMR 8201 - LAMIH, F-59313 Valenciennes, France*
*e-mail: simon.enjalbert@uphf.fr; y.boukal@gmail.com*

**Abstract:** In this work, the problem of reduction of energy consumption based on calculated speed, called eco-driving trajectory for tramway system is investigated, where some constraints on the states, control input and travel time are considered. Thanks to the operating speed range of the tramway, the non-linear model of the system can be approximated by linear one and is given in a state space distance based formulation. The problem studied in this work is to determine the speed profile that minimizes the cost function defined as the energy consumption. Firstly, the constrained optimization problem is formulated as Kuhn-Tucker conditions, which is the general form of the Pontryagin maximum principle, that yield to a local minimum. Then, the control strategy that ensure the efficient consumption for the tramway system is deduced based on the optimal control approach. This leads to the well known driving trajectory for the railway system, which is divided into four phases: acceleration, speed holding, coasting, and braking. On the other hand, the energy consumption analysis allows us to write the cost function as a sum of the kinetic energy to move the train and the resistance forces, by taking into account the gradient and limitations. Based on this analysis,the necessary condition for a is obtained. That ensure method is used to find energy-efficient driving trains. In this paper, we show how to calculate the critical switching points for an optimal strategy non-linear programming algorithm is used to solve the optimal control problem.

*Keywords:* Optimal control problem, Minimization of energy consumption, Non-linear algorithm, Tramway system, Human Machine Systems.

## 1. INTRODUCTION

In the last years, the attention of many researchers has been attracted by the significant increase of railway traffic volumes and their emissions of the pollutant particles. This is one of the reasons for what the issue of speed profiling becomes a new problem with specific characteristics, which must be resolved. Generally, the goal is to optimize the energy consumption or/and running times by involving one or several objectives. For this reason, the question of speed profiling becomes a specific issue, which must be handled in the development phase of eco-aware transportation systems (see Howlett et al. (1993); Coleman et al. (2009); Albrecht et al. (2011); Feng (2011); Gupta et al. (2016) and the references therein). This paper focuses on the continuity of study initiated by Cacciabue et al. (2013); Enjalbert et al. (2013); La Delfa et al. (2016) on the Eco-driving command for tram-driver system where the authors objective was to define a trajectory profile to reduce energy consumption and its impact on the driver vigilance with compliance to the network procedures. This work has led to patent driving assistance device for a railway vehicle by Miglianico et al. (2017). Then it has been complemented to avoid the impact of the driver bad maneuver or behavior in following the tramway system eco-driving reference which affect the system dynamic by Boukal and Enjalbert (2019). In this work, authors will define switching points between the four driving phases: acceleration, speed holding, coasting, and braking. A strategy based on a non-linear programming algorithm is used to solve the optimal control problem. The obtained eco-driving trajectory could be implemented in future human-machine interfaces in tramway cabins.

⋆ This work has been funded by the Project: ELSAT 2020 by CISIT / GS2RI: Greener & Safer Rail Road Interaction.

## 2. MATHEMATICAL MODEL

| | | |
|---|---|---|
| $a_0$, $b_0$, $c_0$ | : | coefficients of the mass, mechanical, |
| | : | air resistances |
| $a_1$, $b_1$ | : | linearised coefficients of the mass, |
| | : | mechanical & air resistances |
| $m$ | : | train mass (Kg) |
| $\beta$ | : | slope angle (°) |
| $r_c$ | : | curvature radius (m) |
| $g$ | : | gravitational constant ($9.81 m.s^{-2}$) |

The dynamic of the tramway is modelled to control the tramway position and velocity at any time during the defined path between two stations. This modelling requires the knowledge of some basic parameters like the traction and brake intensities, adhesion coefficient, track profile, in addition to the weather conditions such as wind speed if available. Usually, the brake and traction curves are given by the constructor. By applying the fundamental Newton's law on the tramway, we obtain its dynamic equation states the relation between the exerted forces on the system, its mass $m$ and the acceleration $\dot{v} = \ddot{p}$

$$m\dot{v} = F_T(v) - F_R(v) \tag{1}$$

where $F_T(v)$ is the traction effort that the tramway produces in the running phase, and $F_R(v)$ represents the resistance which is the sum of the line $F_{Rl}(v)$ , curve $F_{Rc}(v)$ and vehicle $F_{Rm}(v)$ resistances. These later are given as

$$F_{Rl} = mg\sin(\beta) \tag{2a}$$

$$F_{Rc} = mg\frac{k_e}{r_c} \tag{2b}$$

$$F_{Rm}(v) = a_0 + b_0 v + c_0 v^2 \tag{2c}$$

$$F_{Rm}(v) = a_1 + b_1 v \tag{2d}$$

From the equations (2), we can see that the line resistance $F_{Rl}(v)$ depends on the train mass and the slope angle $\beta$, and the gravity constant $g$. Furthermore, the curve resistance $F_{Rc}(v)$ concerns the passage of a tram in some inclined portions of path, and depends on the track gauge coefficient $k_e$ and the curvature radius $r_c$. In addition, due to the low value of $c_0$ and the velocity range $[v_{min}; v_{max}]$ relatively low also, the vehicle resistance $F_{Rm}(v)$ against tram move can be approximated in the following by $a_1 + b_1 v$ by using the least squares method.

### *2.1 Time domain*

The equations of motion for a point mass train are

$$\frac{\mathrm{d}x}{\mathrm{d}t} = v(t) \tag{3a}$$

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{1}{m}u(t) - \frac{b_1}{m}v(t) - \left(\frac{a_1}{m} + g\sin(\beta) + g\frac{k_e}{r_c}\right) \tag{3b}$$

Now, the tramway dynamics can be represented in a state space model as

$$\dot{x}_r(t) = Ax_r(t) + B_u u_r(t) + d_r \tag{4a}$$

$$y_r(t) = C_y x_r(t) \tag{4b}$$

where

$$x_r(t) = \begin{bmatrix} p(t) \\ v(t) \end{bmatrix}, \qquad A = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{b_1}{m} \end{bmatrix}, \qquad B_u = \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}$$

$$C_y = [1 \;\; 0], \qquad d_r = \begin{bmatrix} 0 \\ -\left(\frac{a_1}{m} + g\sin(\beta) + g\frac{k_e}{r_c}\right) \end{bmatrix}$$

### *2.2 Space domain*

From the fact that the track gradient, speed limitations depends on position, it is often convenient to rewrite the equations of motion in the form

$$\frac{\mathrm{d}t}{\mathrm{d}x} = \frac{1}{v(t)} \tag{5a}$$

$$v(t)\frac{\mathrm{d}v}{\mathrm{d}x} = \frac{1}{m}u(t) - \frac{b_1}{m}v(t) - \left(\frac{a_1}{m} + g\sin(\beta) + g\frac{k_e}{r_c}\right) \tag{5b}$$

where the equations (5) are formulated with $x$ as the independent variable. The elapsed time $t = t(x) \in [0, T]$ is the new state space model variable, where $T$ is the total time allowed to travel between two stations. Now the state space variables, which are the speed time $t$ and the speed $v$, are derived with respect to $x$.

## 3. PROBLEM FORMULATION

The power generated by the train is obtained by multiplying the tramway speed by the force delivered by its electric motor. Then, the consumed energy is deduced by integrating the obtained power. We assume, in the energy balance of the tramway, that the energy consumed is equal to the energy delivered by the engine, i.e. the tramway does not recover energy during the braking phase. Based on the energy conservation law, the cost function is deduced from the computation of the total energy consumption, i.e. the energy consumption is considered as the objective function of the considered non-linear programming problem. It will be convenient to cast our optimization problem into the following particular form. This is no a restriction since any optimization problem can be cast into this form

$$\underset{u\in U}{\text{minimize}} \quad \int_0^T (uv)\,\mathrm{d}t \tag{6a}$$

$$\text{subject to} \quad \dot{v} = \frac{1}{m}u - \frac{1}{m}(a_1 + b_1 v + g\frac{k_e}{r_c} + g\sin(\beta(x))) \tag{6b}$$

$$x(0) = 0 \tag{6c}$$

$$x(T) = L \tag{6d}$$

$$v(0) = 0 \tag{6e}$$

$$v(T) = 0 \tag{6f}$$

The control input $u(t)$ is composed of two forces and is given as

$$u(t) = u_f f(v) - u_b b(v) \tag{7}$$

where

- ▷ $u_f$ is the relative traction force, where $u_f \in [0, 1]$;
- ▷ $f(v)$ is the specific maximum traction force;
- ▷ $u_b$ is the relative braking force, where $u_b \in [0, 1]$;
- ▷ $b(v)$ is the specific maximum braking force.

Now, by applying the Pontryagin maximum principle to solve the problem as specified in (6) with its constraints. This leads to find an optimal strategy for optimal control problem. Since the associated Hamiltonian to the problem defined as in (6) is given by

$$H(x, v, \lambda_1, \lambda_2, u) = uv + \lambda_1 v + \lambda_2[\frac{1}{m}u(t) - \frac{b_1}{m}v(t) - (\frac{a_1}{m} + g\sin(\beta) + g\frac{k_e}{r_c})] \tag{8}$$

the adjoin equations are defined as

$$\frac{\mathrm{d}\lambda_1}{\mathrm{d}t} = -\frac{\partial H}{\partial x} = -\lambda_2 g\frac{\mathrm{d}\sin(\beta)}{\mathrm{d}x} \tag{9a}$$

$$\frac{\mathrm{d}\lambda_2}{\mathrm{d}t} = -\frac{\partial H}{\partial v} = -u - \lambda_1 - \lambda_2\left(-\frac{b_1}{m}\right) \tag{9b}$$

Furthermore, we know that the Hamiltonian is constant along the trajectory produced by the optimal control. Despite the fact that the controller of the speed holding is a singular one, it is still the key to the optimal control strategy. From the Hamiltonian (8), it is easy to see that

$$\frac{\partial H}{\partial u} = v + \lambda_2\frac{1}{m} \tag{10}$$

In fact, the optimal control strategy of the problem (6) have one of the following five possible optimal driving modes (Liu and Golovitcher (2003))

- Full power (FP). $u_f = 1$, $u_b = 0$, exists when $\lambda_2 > mv$.
- Partial power (PP). $u_f$ may vary, $u_b = 0$, exists when $\lambda_2 = mv > 0$.
- Inertia motion or coasting (C). $u_f = 0$, $u_b = 0$, exists when $0 < \lambda_2 < mv$.
- Partial braking (PB). $u_f = 0$, $u_b$ may vary, exists when $\lambda_2 = 0$.
- Full braking (FB). $u_f = 0$, $u_b = 1$ , exists when $\lambda_2 < 0$.

*Remark 1.* The PB driving mode is used by the driver in case of unpredictable noise models an unusual situation. This case is not discussed in this work.

According to the obtained solution of the optimization problem, the driving strategy is energy-efficient if it respects the five possible optimal driving modes given previously. Instead of solving the optimization problem given below to ensure the minimal energy consumption, the problem can be reformulated as find the switching points that guarantee the same result, i.e. to be energy-efficient. In this new formulation, the objective function is evaluated and minimized with respect to the new decision variables and constraints, where the considered decision variables are the switching speeds and the duration of each phase. In fact, solving the the optimization problem given as a non-linear programming model leads to obtaining a speed trajectory with less energy consumption. Due to a small number of decision variables in comparison with other approaches, this new formulation of the optimization problem is easy to solve. As a result, the speed reference trajectory is obtained from the decision variables, within a few seconds, making the method fairly suitable for real-time implementation, as detailed in what follows.

*Assumption 1.* We assume that the mass $m$ is constant during the trip between two stations.

## 4. MAIN RESULTS

The main idea is to develop and solve an optimal control strategy to minimize the energy consumption of the tramway by focusing on the optimization of profile of the tramway speed referred as "Eco-Driving", where the different road conditions, including the effects of road gradients and their variations, is also taken into consideration in the optimization problem.

### *4.1 One phase optimal control model*

The proposed optimal problem is obtained by considering the tramway kinetic energy on different speed profile section, i.e. the work needed to accelerate the tramway from rest to its stated velocity, and to maintain certain desired speeds.

Now, we consider the possible driving modes from the optimal speed trajectory in figure 1 where each point $P_i$ is characterized by a spatial localization $x_i$, a speed $v_i$, and the passage time $t_i$. In addition, for each mode $i$ or section, the transit time $\Delta T$ is given as $\Delta T_i = t_{i+1} - t_i$, and the travelled distance $\Delta X$ is obtain also as $\Delta X_i = x_{i+1} - x_i$ .

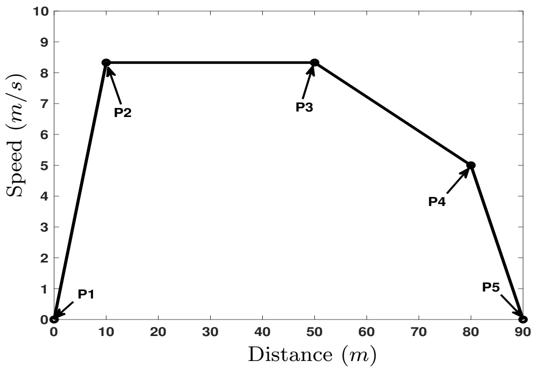


Fig. 1. The four phases of tramway driving: acceleration, speed holding, coasting, and braking

- Full power (FP) $P_1 \to P_2$:

$$E_{FP_i} = \frac{1}{2}v_1^2 = 0 \tag{11a}$$
$$E_{FP_f} = \frac{1}{2}v_2^2 + g\Delta h_{FP} \tag{11b}$$
$$E_{FP} = (E_{FP_f} - E_{FP_i}) + E_{R_{FP}} \tag{11c}$$
$$v_1 = 0 \tag{11d}$$
$$\Delta T_{FP} = \int_{v_1}^{v_2} \frac{\mathrm{d}x}{\frac{1}{m}u - \frac{1}{m}(a_1 + b_1 v + g\frac{k_e}{r_c} + g\sin(\beta(x)))} \tag{11e}$$
$$\Delta X_{FP} = \int_{v_1}^{v_2} \frac{v.\mathrm{d}x}{\frac{v}{m}u - \frac{1}{m}(a_1 + b_1 v + g\frac{k_e}{r_c} + g\sin(\beta(x)))} \tag{11f}$$

- Partial power (PP)$P_2 \to P_3$:

$$E_{PP_i} = \frac{1}{2}v_2^2 \tag{12a}$$
$$E_{PP_f} = \frac{1}{2}v_3^2 + g\Delta h_{PP} \tag{12b}$$
$$E_{PP} = (E_{PP_f} - E_{PP_i}) + E_{R_{PP}} \tag{12c}$$
$$\Delta X_{PP} = \Delta T_{PP}.v_2 \tag{12d}$$
$$v_2 = v_3 \tag{12e}$$

- Inertia motion or coasting (C) $P_3 \to P_4$:

$$E_{C_i} = \frac{1}{2}v_3^2 \tag{13a}$$
$$E_{C_f} = \frac{1}{2}v_4^2 + g\Delta h_C \tag{13b}$$
$$E_C = (E_{C_f} - E_{C_i}) + E_{R_C} \tag{13c}$$
$$\Delta T_C = \int_{v_3}^{v_4} \frac{1}{-\frac{1}{m}(a_0 + b_0 v + g\frac{k_e}{r_c} + g\sin(\beta(x)))}\,\mathrm{d}v \tag{13d}$$
$$\Delta X_C = \int_{v_3}^{v_4} \frac{v}{-\frac{1}{m}(a_0 + b_0 v + g\frac{k_e}{r_c} + g\sin(\beta(x)))}\,\mathrm{d}v \tag{13e}$$

- Full braking (FB) $P_4 \to P_5$:

$$E_{FB_i} = \frac{1}{2}v_4^2 \quad (14a)$$

$$E_{FB_f} = \frac{1}{2}v_5^2 + g\Delta h_C \quad (14b)$$

$$E_{FB} = (E_{FB_f} - E_{FB_i}) + E_{R_{FB}} \quad (14c)$$

$$\Delta T_{FB} = \frac{v_4}{b} \quad (14d)$$

$$\Delta X_{FB} = \frac{v_4^2}{b} \quad (14e)$$

The following parameters $v_2$, $v_4$ and $\Delta T_{PP}$ are considered as the decision variables in the optimization problem to be solved, where the cost function is equal the total energy consumption defined by adding the energy consummated during the first (Full power) and second (Partial power) modes:

$$E = E_{FP} + E_{PP} \quad (15)$$

*Assumption 2.* We assume that no energy is consumed nor recovered during braking, i.e $E_{FB} = 0$.

Notice that, the total travelled distance is equal to the distance between two stations, and is equal also the sum of the distances of the phases. Similarly, the total running time is linked to the travel time of each phase, and is equal to the required one.

$$\sum_{i=1}^{4} \Delta T_i = T \quad (16a)$$

$$\sum_{i=1}^{4} \Delta X_i = L \quad (16b)$$

In the following theorem, we provide new formulation for the tramway speed profile optimization as a non-linear programming problem ensuring the minimization of the energy consumption, i.e. the energy-efficiency.

*Theorem 1.* The higher energy efficiency of a tramway is ensured if the optimization problem given below

$$\underset{u\in U}{\text{minimize}} \quad E = E_{FP} + E_{PP} \quad (17a)$$

$$E = \frac{1}{2}v_2^2 + \underbrace{g\Delta h_{FP} + g\Delta h_{PP}}_{g\Delta h} + \underbrace{E_{R_{FP}} + E_{R_{PP}}}_{E_R} \quad (17b)$$

$$\text{subject to} \quad \sum_{i=1}^{4} \Delta T_i = T \quad (17c)$$

$$\sum_{i=1}^{4} \Delta X_i = L \quad (17d)$$

$$\Delta X_i > 0,\ i = 1, \dots, 4. \quad (17e)$$

$$\Delta T_i > 0,\ i = 1, \dots, 4. \quad (17f)$$

$$v_i > 0,\ i = 2, 4. \quad (17g)$$

has a solution, where the decision variables of this non-linear programming model are $v_2$, $v_4$ and $\Delta T_2$. Then, the total energy consumption per mass unit is minimized by following the speed trajectory defined by $v_2$, $v_4$ and $\Delta T_2$.

The presented optimal control problem is less complex, this comes form the fact no partial differential equations are involved to solve it. The switching points, indicating that a train must reach the position $P_i$ at the specified time $t$ and speed $v$, can be deduced from the solution of the optimal control problem. Consequently, which allow us to identify the complete optimal control sequence. These switching points and the optimal controls represent the complete solution for the optimal control problem.

### *4.2 Multiple-phase optimal control model*

It is not possible to solve the optimal control problem to achieve energy efficiency from the cost function, dynamic constraints, path constraints and boundary constraints form the optimal control problem given by (18) in sub-section 4.1, since the path constraints (the speed limits) and the dynamic constraints (gradients) change along the tramway trajectory. So, the considered optimal control problem can be represented as multiple-section optimal control problems. To achieve that, the trajectory is divided into sections, where any section has its own cost function, dynamic model, path constraints, and boundary conditions. Firstly, let $T_i = \sum_{j=1}^{4} \Delta T_{ij}$ denotes the travel time of each section $k$. In fact, the total run time between a station $i$ and the next one $i+1$ is denoted by $T$, where $T$ is equal the sum of the travel time $T_i$ of all sections, i.e. $T = \sum_{i=1}^{n} \sum_{j=1}^{4} \Delta T_{ij}$. In addition, the total run time is equal to the sum of a fixed value $T_{i_f}$ and a value $T_{i_v}$ that must be minimized, i.e. $T_i = T_{i_f} + T_{i_v}$. The variable $T_{i_v}$ will be used to relax the optimization problem. On the other hand, the terms $S_i = \sum_{j=1}^{4} \Delta S_{ij}$ denotes the travel distance of each section $k$. In fact, the total distance between a station $i$ and the next one $i+1$ is denoted by $S$, where $S$ is equal the sum of the travel distances $S_i$ of all sections, i.e. $S = \sum_{i=1}^{n} \sum_{j=1}^{4} \Delta S_{ij}$. In this case, the optimal solution is obtained by assembling the all phases of trajectory sections. The advantages presented by this approach are the speed limits and gradient are constant in each section. In addition, the speed $v_5$ of a section $i$ is equal to $v_1$ of the section $i+1$, this boundary conditions link the sections together. The total energy consumption is given by summing the cost functions of all phases. Finally, the optimal trajectory is obtained by solving the minimization problem defined by new total cost function subject to new constraints depending on limits and gradients of each section.

*Theorem 2.* The higher energy efficiency of a tramway is ensured if the optimization problem given below

$$\underset{u\in U}{\text{minimize}} \quad E = \sum_{i=1}^{n} E_i \quad (18a)$$

$$E_i = \frac{1}{2}v_{i2}^2 + \underbrace{g\Delta h_{iFP} + g\Delta h_{iPP}}_{g\Delta h_i} + \underbrace{E_{R_{iFP}} + E_{R_{iPP}}}_{E_{iR}} \quad (18b)$$

$$\text{subject to} \quad \sum_{i=1}^{n} \sum_{j=1}^{4} \Delta T_{ij} = T \quad (18c)$$

$$\sum_{i=1}^{n} \sum_{j=1}^{4} \Delta X_{ij} = L \quad (18d)$$

$$\Delta X_{ij} > 0,\ i = 1, \dots, n; \quad j = 1, \dots, 4 \quad (18e)$$

$$\Delta T_{ij} > 0,\ i = 1, \dots, n; \quad j = 1, \dots, 4 \quad (18f)$$

$$v_{ij} > 0,\ i = 1, \dots, n; \quad j = 2, 4 \quad (18g)$$

has a solution, where the decision variables of this non-linear programming model are $v_i2$, $v_i4$ and $\Delta T_i2$. Then, the total energy consumption per mass unit is minimized by following the speed trajectory defined by $v_i 2$, $v_i 4$ and $\Delta T_i 2$.

## 5. CASE STUDY

In this part, numerical simulations are presented to illustrate the effectiveness of the proposed methodology. Firstly, different study cases are considered, where the speed limitations are arranged in different order, sometimes increasing, decreasing, or a varied order between increasing and decreasing. The traffic on these lines is provided by Alstom Citadis type 302 trams. The mechanical characteristics of the considered model and line geometry have been parametrized and introduced in the simulated model. A comparison between energy consumption of fast, average and slow times is made. In fact, the fast travel time is obtained by rounding the 105% of the minimal time in multiples of $5s$. Also, by rounding the fast time by 110% in multiples of $5s$, we get average travel time. Finally, the slow travel time is obtained by adding the difference between the average and the fast time to the average time, i.e. $T_f - T_s = 2 \times (T_a - T_f)$ where $T_f$ is the fast time, $T_a$ is the average time and $T_s$ is the slow time.

### *5.1 Increasing limits*

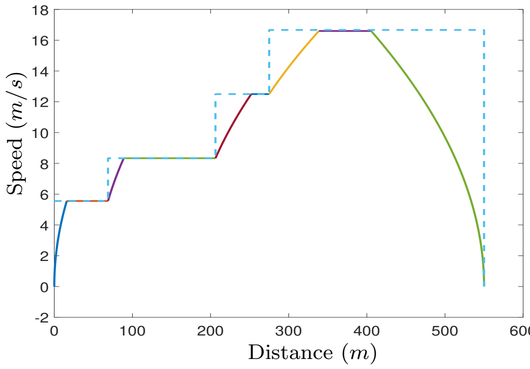


Fig. 2. The speed trajectory of the minimal trip time and his characteristics $T_i = 64.4$, $E = 1.0257e^7$

In this scenario, the consumption energy used during a trip with an increasing speed limits is studied. From the solution of the optimization problem, the switching points and the duration of partial power phase are deduced for different trip time constraints. In fact, the speed trajectories can be drawn by using these informations. As a result, the speed trajectory of the minimal trip time is plotted in the figure 2 for a travel distance $S_i = 550m$. In addition, the speed trajectories of the fast and slow trip times are plotted also in figures 3, and 4 respectively.

From figures 3-4, we can remark that, by lengthening the trip time, the reached maximum speed is less than the imposed speed limitation. In addition, the coasting phase becomes more solicited. This leads to minimize the energy used to make the make the trip.

### *5.2 Decreasing limits*

In this scenario, the consumption energy used during a trip with a decreasing speed limits is studied. From the solution of the optimization problem, the switching points and the duration of partial power phase are deduced for different trip time constraints. In fact, the speed trajectories can be drawn by using these informations. As a result, the speed trajectory of the minimal trip time is plotted in the figure 5 for a travel distance $S_i = 550m$.

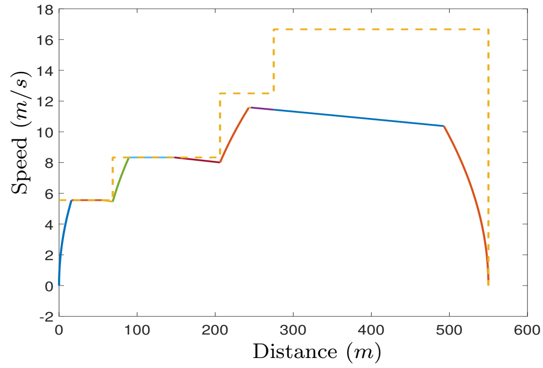


Fig. 3. The speed trajectory of the fast trip time and his characteristics $T_i = 70$, $E = 4.7801e^6$

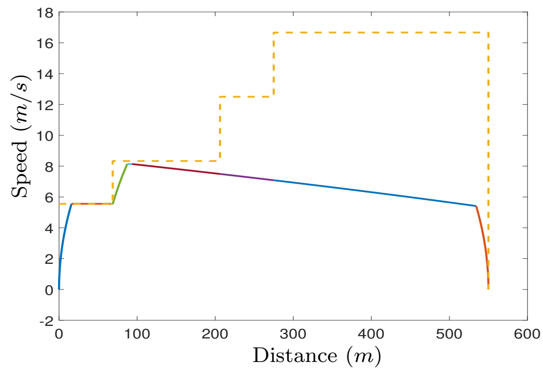


Fig. 4. The speed trajectory of the slow trip time and his characteristics $T_i = 90$, $E = 2.2577e^6$

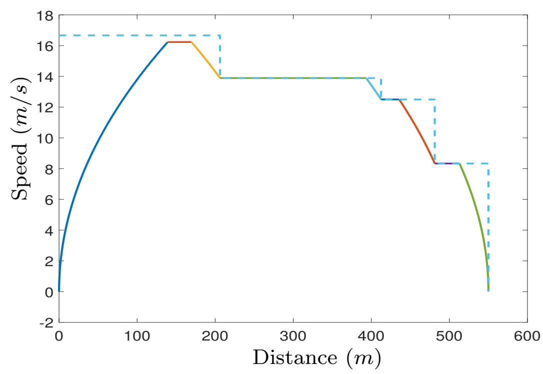


Fig. 5. The speed trajectory of the minimal trip time and his characteristics $T_i = 55.2$, $E = 1.0348e^7$

### *5.3 Varied order*

In this scenario, the consumption energy used during a trip with a varied order of speed limits is studied. From the solution of the optimization problem, the switching points and the duration of partial power phase are deduced for different trip time constraints. In fact, the speed

| Type | Fast | Average | Slow |
|---|---|---|---|
| Increasing | $E_f = 4.78e^6$ | $E_a = 3.15e^6$ -34.05% | $E_s = 2.26e^6$ -52.77% |
| Decreasing | $E_f = 4.80e^6$ | $E_a = 3.15e^6$ -34.3% | $E_s = 2.43e^6$ -49.29% |
| Varied | $E_f = 4.07e^6$ | $E_a = 3.16e^6$ -22.35% | $E_s = 2.74e^6$ -32.69% |

Table 1. The energy consumption for fast, average and slow times

trajectories can be drawn by using these informations. As a result, the speed trajectory of the minimal trip time is plotted in the figure 6 for a travel distance $S_i = 550m$.

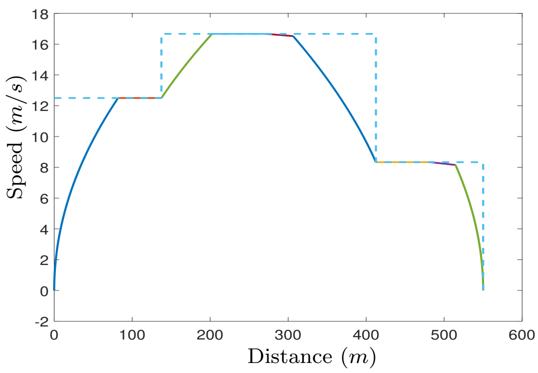


Fig. 6. The speed trajectory of the minimal trip time and his characteristics $T_i = 57.74$, $E = 1.0024e^7$

From Table 1 where energy results and associated savings for all cases studied are merged, it is clear that lengthening the trip time saves up to 50 percent of the energy consumed during the fast time. Consequently, this fact confirms that lengthening the imposed trip time saves the consumed energy.

## 6. CONCLUSION

In this paper, the control strategy to reduce energy consumption for the Tramway system is deduced based on the optimal control approach. The calculation of the critical switching points between acceleration, speed holding, coasting, and braking phases is realized thanks to the presented non-linear programming algorithm. The perspective of this work is to simulate the proposed methodology on The PSCHITT platform (Hybrid and Inter-modal Collaborative Simulation Platform in Land Transport). The PSCHITT platform is a versatile simulator that can be fitted with different cabins (Reduced Mobility Person, Rail, ...) according to the scientific objectives and experimental needs, for example PSCHITT-Rail [1]. Cases based on the commercial tramway of Valenciennes city in France which is composed of two lines, with about $33.8km$ of tracks and 48 stations will be studied. The evaluation of the impact on traffic for several tramways operating on the same time in a city network and the concept of mobility resilience based on previous work on the domain of transportation (see Enjalbert et al. (2011); Enjalbert and Vanderhaegen (2017)) should be enhanced.

[1] https://www.uphf.fr/LAMIH/en/PSCHITT-Rail


## ACKNOWLEDGEMENTS

The present research work has been supported by:
The European Community,
The Ministry of Higher Education and Research,
And The Hauts-de-France Region.
The authors gratefully acknowledge the support of these institutions.


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