Analytical model for the energy harvesting of a spherical sensor from ambient vibrations

Daher Diab, Fabrice Lefebvre, Georges Nassar, Nikolay Smagin, Adnan Naja,
Fawaz El Omar

Analytical model for the energy harvesting of a spherical sensor from ambient vibrations

D. Diab, F. Lefebvre, G. Nassar, N. Smagin

Department of Opto-Acousto-Electronics
Institute of Electronics, Microelectronics and
Nanotechnology UMR CNRS 8520
59313 Valenciennes Cedex 9, France
Daher.Diab@etu.univ-valenciennes.fr

A. Naja, F. El Omar

Laboratory of Physics and Modeling, EDST, Lebanese
University
1300 - Tripoli, Lebanon

Abstract— In this work an analytical model for the energy harvesting of an acoustic spherical sensor has been developed in the context to make it autonomous. Our spherical sensor is composed of two half-spheres of Plexiglas and a piezoelectric ring of PZ26 that can be used as exciter or sensor. For the analytical model, the piezoelectric ring was modeled using two primary modes of vibration: thickness and radial. For each mode, the ring is described by an equivalent electromechanical model which connects the mechanical part (forces and velocities) to the electrical part (voltage and current). The proposed paper theoretical model enables building a global electromechanical circuit in order to simulate the total harvested voltage response.

Keywords—energy harvesting; piezoelectric ring; spherical sensor; vibration modes

I. INTRODUCTION

In recent years, much research has been done on the issue of energy autonomy and in particularly on the supply of sensors. The energy harvesting is a theme devoted to the use of the ambient energy (vibration, light, and temperature) present in the environment for powering electronic devices (Sensors, mobile equipment), in a way to extend their operating life and make them completely autonomous.

Ambient energy sources are numerous, and from this, ambient mechanical vibrations are very studied and starting to be used. In a general way, vibratory energy recovery are based on resonant mechanical systems (spring-mass-damper) tuned to the frequency of the source [1]. Several physical phenomena are used for extraction of mechanical energy and its conversion into electrical energy, the piezoelectricity [2], the electrostatic [3] and the electromagnetism [4] can be used as an example of the conversion phenomena. In the context of our work, we focused particularly on piezoelectricity.

A device for recovering energy from ambient vibrations consists essentially of four necessary units: The first unit is a purely mechanical device its role is to capture and optimized mechanical vibrations, The second is an electromechanical device whose purpose is to transform the recovered mechanical energy into electrical energy, The third unit is an electrical device or more precisely an electrical circuit capable of converting the non-exploitable electric energy into exploitable electrical energy, and the last unit is a device for storing the energy before being applied to the sensor to make it electrically

autonomous. To improve the power density of the generators and their bandwidth, it is necessary to optimize the four conversion units.

II. SENSOR DESCRIPTION

In this work a spherical device as an acoustic sensor has been developed. This sensor is composed of a closed spherical shell, assembled from two half-spheres made of Plexiglas. Its outer radius is 11 mm and the inner radius is 10 mm. An active element (piezoelectric ring) is sandwiched between the two half spheres [5]. The electronic embarking the sensor application is placed in the hollow portion of the sphere. It ensures the excitation of the piezoelectric ring, the control of the transceiver, the processing and data storage (fig. 1).

Fig. 1. The hollow portion with the embedded electronics.
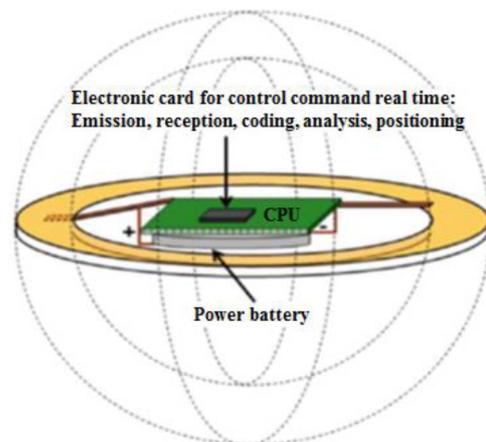
The hollow portion contains a programmable chip (SoC) which ensures the control of the resonator. It includes a central processing unit (CPU), a digital oscillator (DCO), a task scheduler (IT), an encoded ultrasonic generator (PWMG), a power management unit (PMU), an output amplifier (OB), and a battery (BAT).

## III. MODELIZATION OF THE PIEZOELECTRIC RING

The harvesting of vibrational energy by piezoelectric transduction has a great interest. Recently, many researchers [6, 7], have worked on the development of systems based on different conversion techniques. In the case of our sensor, the

conversion of the vibrational energy into electric energy is achieved by means of a piezoelectric ring undergoing deformations exerted by a sphere excited by external vibrations. For this purpose, the paper models the piezoelectric ring according to its two main modes of vibration: thickness and radial.

A. Thickness mode

Consider an elementary slice of the ring, thickness dz and section A. The displacements u and the forces F are along the longitudinal axis (axis z) as indicated in the figure 2 [8]:

Fig. 2. Elementary slice of the ring in the thickness oscillation
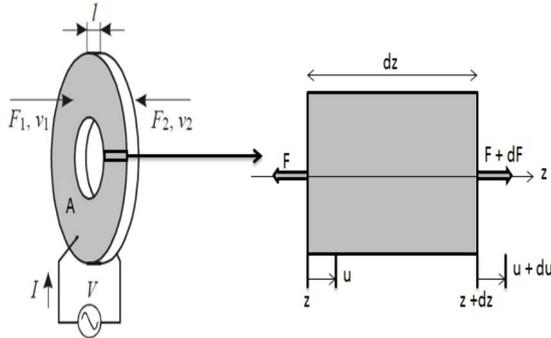
The fundamental principle of dynamics applied to this elementary volume gives [8]:

$$\rho . A . dz \frac{\partial^2 u}{\partial t^2} = \frac{\partial F}{\partial z} dz \qquad (1)$$

The constitutive equations of a passive solid material are:

$$T = c_{33} . S \quad T = F/A \quad S = \partial u / \partial x \qquad (2)$$

The combination of these equations gives:

$$\rho . \frac{\partial^2 u}{\partial t^2} = c_{33} . \frac{\partial^2 u}{\partial z^2} \qquad (3)$$

With ρ is the density of the PZ26 and c_{33} is its Young's modulus.

Assuming the harmonic vibrations and using the complex notation a way of separating the variables $u(z, t) = U(z)e^{j\omega t}$, the equation 3 becomes:

$$\frac{\partial^2 U}{\partial z^2} + \omega^2 . \frac{\rho}{c_{33}} . U = 0 \qquad (4)$$

The solutions of this equation are given by:

$$U(z) = \alpha . e^{-jkz} + \beta . e^{jkz} \qquad (5)$$

Where $k = \omega . \sqrt{\frac{\rho}{c_{33}}}$ is the wave vector in m-1, α and β are two constants.

The ring has a thickness L, the values of the coefficients α and β depends to the boundary conditions on the faces of the ring and supposing t = 0:

$$\dot{u}(0,0) = v_1 = j\omega (\alpha + \beta) \qquad (6)$$

$$\dot{u}(L,0) = v_2 = j\omega (\alpha . e^{-jkL} + \beta . e^{jkL}) \qquad (7)$$

This gives:

$$\alpha = \frac{v_2 - v_1 e^{jkL}}{2\omega \sin(kL)} \text{ and } \beta = \frac{v_2 + v_1 e^{-jkL}}{2\omega \sin(kL)}$$

The forces at the ring ends z = 0 and z = L are calculated by:

$$F(0) = F_1 = Z \left[\frac{(v_1 + v_2)}{j \sin(kL)} + j \tan \left(\frac{kL}{2}\right) v_1 \right] \qquad (8)$$

$$F(L) = F_2 = Z \left[\frac{(v_1 + v_2)}{j \sin(kL)} + j \tan \left(\frac{kL}{2}\right) v_2 \right] \qquad (9)$$

Where Z is the acoustic impedance of the ring; it is expressed as a function of c_{33} and the density ρ by:

$$Z = F/\dot{u} = A . \sqrt{c_{33} . \rho} \qquad (10)$$

If take into account the piezoelectricity of the ring, the expression of the constraint has to be modified according to the constitutive equations of the piezoelectric material:

$$T = C^D . S - h_{33} . D \qquad (11)$$

With $C^D = c_{33} + \frac{e^2}{\varepsilon_{33}^S}$ and $h_{33} = \frac{e}{\varepsilon_{33}^S}$.

So the two forces are calculated replacing T by T + h.D, which give the following system:

$$F_1 - h_{33} DA = Z \left[\frac{(v_1 + v_2)}{j \sin(kL)} + j \tan \left(\frac{kL}{2}\right) v_1 \right] \quad (12)$$

$$F_2 - h_{33} DA = Z \left[\frac{(v_1 + v_2)}{j \sin(kL)} + j \tan \left(\frac{kL}{2}\right) v_2 \right] \quad (13)$$

Note that the term $h_{33} DA$ represents a force denoted F due to the piezoelectric contribution. Now this electromechanical model can be transposed in an electromechanical equivalent circuit as shown the following figure:

Fig. 3. Electromechanical circuit of the piezoelectric ring in the thickness mode
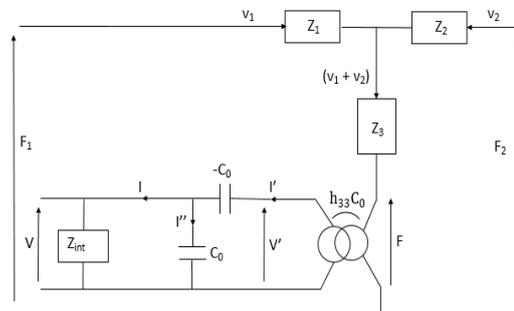
Here, $Z_1 = Z_2 = jZ.\tan(\frac{kL}{2})$, $Z_3 = -j\frac{Z}{\sin(kL)}$ and $C_0 = \frac{\varepsilon_{33}^S A}{L}$ the blocked capacity of the ring.

1) Analytical response in voltage

The electromechanical equivalent circuit is formed of two parts connected by a transformer of ratio $h_{33}C_0$, a pure mechanical part designated by the mechanical forces F_1 and F_2 and the displacement velocities v_1 and v_2, an electric part to the output of the transformer designated by the generated electric voltage and current V and I.

This circuit allows us to write:

$$\overline{F} + Z_3 (v_1 + v_2) = \overline{F}_1 - Z_1 v_1 = \overline{F}_2 - Z_2 v_2 \quad (14)$$

For a symmetric velocity field compared to the piezoelectric ring plane, we can suppose that $v_1 = v_2$.

This gives:

$$\overline{F}_1 = \overline{F} + (2Z_3 + Z_1) v_1 \quad (15)$$

The voltage and current at the output of the transformer are given by:

$$\overline{V}' = \overline{F}/h_{33}.C_0 \quad (16)$$

$$I' = h_{33}.C_0(v_1 + v_2) = 2h_{33}C_0 v_1 \quad (17)$$

$$I'' = I' - I = jC_0\omega\overline{V} \text{ and } I = \overline{V}/Z_{int} \quad (18)$$

From where:

$$\overline{V}\left(1 + \frac{1}{jC_0\omega.Zint}\right) = \frac{I'}{jC_0\omega} = \frac{2h_{33}}{j\omega} v_1 \quad (19)$$

The equivalent impedance of the electrical part is given by:

$$Z_{elec} = \frac{-1}{jC_0\omega} + \frac{Z_{int}}{1 + jC_0\omega.Z_{int}} \quad (20)$$

We have $\overline{V}' = Z_{elec}.I'$ which give the relation between the force F and velocity v_1 which is of the following form:

$$\overline{F} = 2h_{33}^2 C_0{}^2 Z_{elec}.v_1 \quad (21)$$

The equation 15 gives:

$$\overline{F}_1 = (2h_{33}^2 C_0{}^2.Z_{elec} + 2Z_3 + Z_1) v_1 \quad (22)$$

Therefore the equation 18 becomes of the form:

$$\overline{V}\left(1 + \frac{1}{jC_0\omega.Z_{int}}\right) = \frac{2h_{33}}{j\omega} \frac{1}{(2h_{33}^2 C_0{}^2.Z_{elec} + 2Z_3 + Z_1)} \overline{F}_1 \quad (23)$$

Where the voltage $\overline{V}$ versus the force $\overline{F}_1$ is:

$$\overline{V} = \frac{2h_{33}C_0.Z_{int}}{(1 + jC_0\omega.Z_{int})(2h_{33}^2 C_0{}^2.Z_{elec} + 2Z_3 + Z_1)} \overline{F}_1 \quad (24)$$
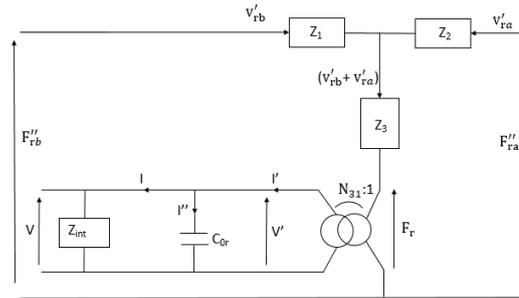
B. Radial mode

To obtain the equations of motion, the constitutive equations of piezoelectricity are written in cylindrical coordinates [9]. In this case the variables are the radius r, the angle θ and the axial dimension denoted z (fig. 4). In this figure, L, a and b are respectively thickness, outer and inner radius of the ring, F_{ra}, F_{rb} and v_{ra}, v_{rb} are external radial forces and vibrational velocities at the outer and inner surfaces of the ring.

Fig. 4. The piezoelectric ring in radial mode
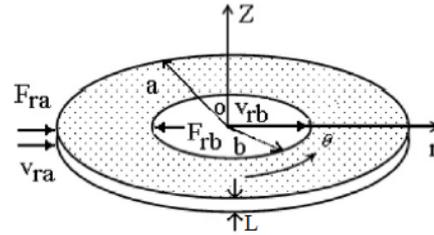
The electromechanical equivalent circuit of the piezoelectric ring in the radial mode is of the following form:

Fig. 5. Electromechanical circuit of the piezoelectric ring in the radial mode

Here, $C_{0r} = \frac{\varepsilon_{33}^T A}{L}\left[1 - \frac{2d_{31}^2}{\varepsilon_{33}^T(s_{11}^E + s_{12}^E)}\right]$ is the blocked capacity of the ring in radial mode and $N_{31} = \pi^2 \alpha ab.\frac{d_{31}}{s_{11}^E + s_{12}^E}$ the electromechanical coupling factor of the ring, s_{ij}^E is elastic compliance constants measured at constant electric field, d_{31} is piezoelectric strain constant, ε_{33}^T is dielectric constant measured at constant stress, $A = \pi(a^2 - b^2)$ is cross-sectional area, $\alpha = \omega/v_r$, $v_r = \sqrt{c_{11}^E/\rho}$ are radial wave number and sound speed, $c_{11}^E = 1/(S_{11}^E(1 - \upsilon_{12}^2))$, with $\upsilon_{12} = -S_{12}^E/S_{11}^E$ is Poisson ratio.

In figure 5, $F''_{ra} = (\pi\alpha b/2) F_{ra}$, $F''_{rb} = (\pi\alpha a/2) F_{rb}$, $v'_{ra} = (2/\pi\alpha b)v_{ra}$, $v'_{rb} = (2/\pi\alpha a)v_{rb}$ and $F_r = N_{31}V$, with $V = E_z L$, E_z is electric field in the z direction.

The expressions of the three impedances are given by:

$$Z_1 = \frac{\pi^2(\alpha a)^2 Z_{rb}}{4j}\left[\frac{Y_1(\alpha a)J_0(\alpha b) - J_1(\alpha a)Y_0(\alpha b)}{J_1(\alpha a)Y_1(\alpha b) - J_1(\alpha b)Y_1(\alpha a)} + \frac{(1 - \upsilon_{12})}{\alpha b}\right] - j\frac{\pi\alpha a Z_{rb}}{2}\frac{1}{J_1(\alpha a)Y_1(\alpha b) - J_1(\alpha b)Y_1(\alpha a)} \quad (25)$$

$$Z_2 = \frac{\pi^2(\alpha b)^2 Z_{ra}}{4j}\left[\frac{Y_1(\alpha b)J_0(\alpha a) - J_1(\alpha b)Y_0(\alpha a)}{J_1(\alpha a)Y_1(\alpha b) - J_1(\alpha b)Y_1(\alpha a)} - \frac{(1-\upsilon_{12})}{\alpha a}\right] - j\frac{\pi\alpha b Z_{ra}}{2}\frac{1}{J_1(\alpha a)Y_1(\alpha b) - J_1(\alpha b)Y_1(\alpha a)} \quad (26)$$

$$Z_3 = j\frac{\pi\alpha a Z_{rb}}{2}\frac{1}{J_1(\alpha a)Y_1(\alpha b) - J_1(\alpha b)Y_1(\alpha a)} = j\frac{\pi\alpha b Z_{ra}}{2}\frac{1}{J_1(\alpha a)Y_1(\alpha b) - J_1(\alpha b)Y_1(\alpha a)} \quad (27)$$

In these equations, J_0, Y_0, and J_1, Y_1 are Bessel functions of order zero and one respectively, $Z_{ra} = \rho v_r S_a$, $Z_{rb} = \rho v_r S_b$, with $S_a = 2\pi a L$ and $S_b = 2\pi b L$ are outer and inner surfaces of the ring respectively.

The analytical voltage response of this vibration mode calculated from the equivalent circuit is expressed as the following form:

$$\overline{V} = \frac{2N_{31}.Z_{int}}{(1 + jC_{0r}\omega.Z_{int})(2N_{31}^2.Z_{elec} + 2Z_3 + Z_2)}\overline{F''}_{ra} \quad (28)$$

Here, $Z_{elec} = Z_{int} / (1 + jC_{0r}\omega.Z_{int})$ is the equivalent impedance of the electrical part.

C. Internal load of energy extraction circuit

The two equivalent electromechanical circuits for the two modes of vibration are completed with the internal impedance load (Z_{int}) of the processing circuit of the harvested energy. So to consider this charge effect, the power transferred has been modeled depending on the internal impedance of the energy extraction circuit.

The shape of the power (fig. 6) shows that the power reaches a maximum value for $Z_{int} = 24$ KΩ. This optimal value of the load will allow a recovery of maximum power of the vibrational radial mode.

Fig. 6.   Power modeled depending of the internal impedance
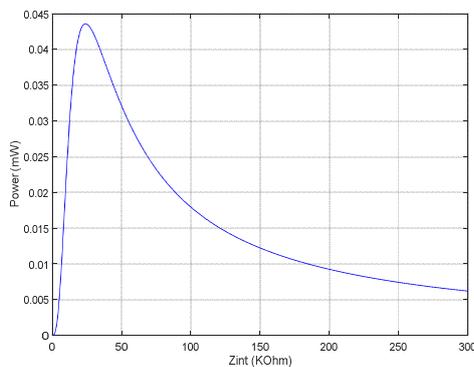
In a same manner, we modeled the power for the longitudinal mode of vibration, the impedance of load obtained are shown in the following table:

TABLE I.   INTERNAL IMPEDANCES FOR THE TWO VIBRATIONAL MODES

| Mode	Longitudinal	Radial
Z_{int} (KΩ)	28	24

So for the future, we used the internal load impedance of the radial vibration mode (24 KΩ) to determine the total harvested voltage given by the two modes of vibration because the radial mode gives the highest voltage by comparison with the longitudinal mode.

II. ANALYTICAL MODEL OF THE ENERGY HARVESTING
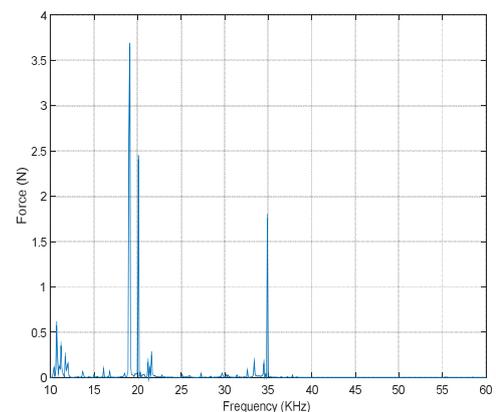
A. Forces field applied to the ring

When using our sensor in a flowing fluid, the sphere is deformed under the action of the stresses produced by the fluid; the deformation of the structure creates stresses on the piezoelectric ring. Therefore to calculate the force field applied to the ring due of the fluid flow, a finite element model was built under COMSOL Multiphysics (fig. 7).

Fig. 7.   Mesh of the sensor showing the ring zone
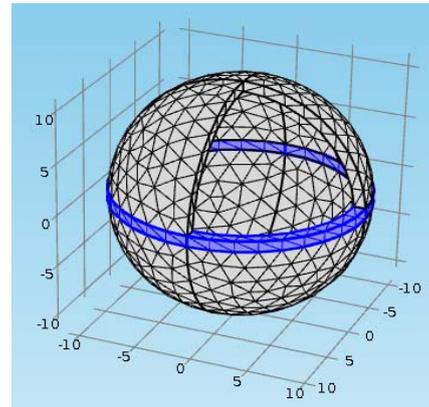
This model makes it easy to combine three approaches: acoustic, mechanical and piezoelectric conversion using three modules "Pressure Acoustics", "Solid Mechanics" and "Electrostatics". So from this model we determined the sound pressure per unit area applied in the plane of the piezoelectric ring which varies depending on the frequency. The integration of this pressure on the entire ring surface gives the force applied on the faces of the piezoelectric ring placed at the junction of the two half spheres (fig. 8).

Fig. 8.   Force applied in the ring plane

This figure shows the shape of the force field applied to the ring for the frequency band between 10 and 60 KHz. This force

is then applied as data input to the two vibrational models of the piezoelectric ring.

B. Proposed analytical model

The objective of our theoretical modeling is to assemble the two electromechanical circuits corresponding to the two modes of vibration in a way to simulate the total recovered energy treated by a specific electronics and stored in a capacitor (fig. 9). The entire modeling must be subsequently made under Spice software which is able to simulate electronic circuits.

Fig. 9.   Theoretical model of the two-mode simulation
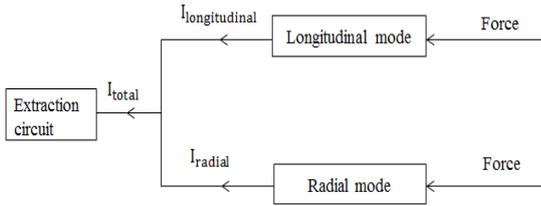
This model has a pure mechanical input designated by the mechanical forces exerted on the piezoelectric ring of our spherical sensor; these forces are described in IV.A. This analytical model given as output the electrical voltage and current generated by the two electromechanical diagrams. So to determine the total recovered voltage, it is necessary to model the two modes of vibration using the same internal load impedance, and in this case we have proposed to use the impedance of the radial mode (24 KΩ) because this last generates the highest voltage. So the total voltage produced by our theoretical model is only the sum of the two voltages given by the two modes of vibration.

$$V_{total} = V_{longitudinal} + V_{radial} \qquad (29)$$

A Matlab script has been written to implement the analytical equation 29, and calculate the total voltage response of our proposed model in the frequency range from 10 kHz up to 60 kHz. The obtained results are shown in the following figure:

Fig. 10. Analytical voltage response of our theoretical model
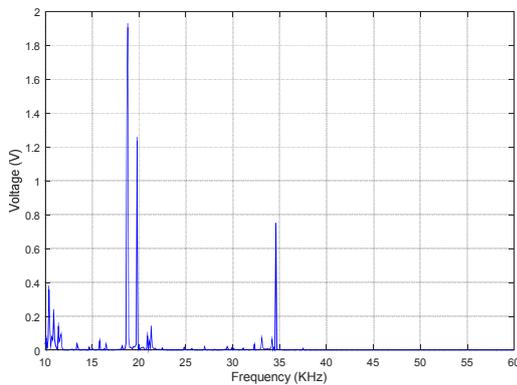
III. Conclusion

In this study, we sought to develop an analytical model for the recovery of vibrational energy of a spherical sensor. This sensor consists of a piezoelectric ring sandwiched between two half-spheres. This model must produce the electromechanical conversion potential of such a sensor according to its two main modes of vibration. For each mode, an equivalent circuit has been established to be integrated with the Spice simulation model of the energy processing circuit. A finite element model was constructed under COMSOL Multiphysics to determine the input force for the two vibrations modes. The experimental validation of this approach is currently underway.

References

[1] Cottone, F., Introduction to vibration energy harvesting. NiPS Energy Harvesting Summer School, 2011.J. Clerk Maxwell, A Treatise on Electricity and Magnetism, 3rd ed., vol. 2. Oxford: Clarendon, 1892, pp.68–73.

[2] Ajitsaria, J., Modeling and analysis of a bimorph piezoelectric cantilever beam for voltage generation. April 2007.K. Elissa, "Title of paper if known," unpublished.

[3] S. Meninger, Vibration to electric energy conversion. IEEE Transactions on Very Large Scale Integration (VLSI), February 2001.

[4] Glynne-Jones, P., An electromagnetic, vibration powered generator for intelligent sensor systems. February 2004.

[5] Medjdoub, A., Study and conception of a spherical acoustic sensor, miniaturized, encoded and autonomous. may 2016.

[6] Sainthuile, T., Energy Harvesting Process Modelling of an Aeronautical Structural Health Monitoring System Using a Bond-Graph Approach. International Journal of Aerospace Sciences, 2012.

[7] Erturk, A., An experimentally validated bimorph cantilever model for piezoelectric energy harvesting from base excitations. IOPscience, 13 January 2009.

[8] Lin, S., Study on the sandwich piezoelectric ceramic ultrasonic transducer in thickness vibration. IOP science 8 January 2008.

[9] Lin, S., Electro-mechanical equivalent circuit of a piezoelectric ceramic thin circular ring in radial vibration. July 2006.

Appendix

TABLE II.   Plexiglas Parameters Used In The Simulation Of The Spherical Shell

Elasticity module E (MPa)	Poisson coefficient ν	Density ρ (kg/m³)
3300	0.39	1190

TABLE III.   Pz26 Parameters Used In The Simulation Of The Piezoelectric Ring

ε_0(F/m)	$\varepsilon^S_{33}/\varepsilon_0$	$\varepsilon^T_{33}/\varepsilon_0$	ρ(kg/m³)	h_{33}(V/m)	e_{33}(C/m²)
8.8×10^{-12}	700	1300	7700	23.7×10^8	14.7
d_{31}(C/N)	c^D_{33}(N/m²)	c^E_{11}(N/m²)	s^D_{33}(m²/N)	s^E_{11}(m²/N)	s^E_{12}(m²/N)
-130×10^{-12}	9.56×10^{10}	7.7×10^{10}	11×10^{-12}	13×10^{-12}	-4.35×10^{-12}