Fuzzy adaptive backstepping sliding mode control of the cart-pendulum system

Khalissa Behih, Djamila Zehar, Khier Benmahammed, Abdelmalik Taleb-Ahmed

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Behih Khalissa[1], Zehar Djamila[2]
[1,2]: Electromechanic department, University of BBA.
[1,2]:Electrotechnic department, University of Setif.
[1,2]: ALGERIA
[1]: khalissabehih@yahoo.fr
[2]: z.dj2005@hotmail.com

Benmahammed Khier[3], Abdelmalik Taleb Ahmed[4]
[3]: LSI Laboratoty University of Setif.
[4]: LAMIH UMR CNRS 8201, University of Valenciennes and of Hainaut Cambrésis
[3]: ALGERIA, [4]: FRANCE
[3] : khierben@gmail.com
[4] : taleb@univ-valenciennes.fr

***Abstract*—A novel adaptive backstepping sliding mode control (ABSMC) law with fuzzy monitoring strategy is proposed for the tracking control of a cart-pendulum system. The proposed ABSMC scheme combining the sliding mode control and the backstepping technique, ensure that the occurrence of the sliding motion in finite time and the trajectory of the tracking error converge to equilibrium point. Furthermore, we introduce fuzzy monitoring strategy to approximate the unknown nonlinear functions of the system model and moreover to approximate the switching control term of the sliding control in order to resolve the chattering problem. Theconvergenceand stability of the proposed control scheme are proved using Lyapunov's method. Finally many simulation results for the cart-Pendulum system are given to illustrate the good tracking performances.**

Keywords—Backstepping; Sliding mode control; Nonlinear uncertain system; Fuzzy adaptive control; Lyapunov stability

I. Introduction

Controller for nonlinear systems, are widely used and implemented in the industry, in order to improve their performances, however there exist external disturbance, parameter variations, and system uncertainty in harsh environment, which consequently degrade the performance of the control system.Various nonlinear control methods have been proposed for solving this problem, including sliding mode control (SMC), backstepping control, intelligent control [5], etc. In the SMC control methodology, the controller switches between two structures to bring the system states to a previously defined sliding manifold [12]. In the design process of the controller, a Lyapunov function is defined which is used to derive the stability of the system. Also the SMC can provide faster dynamics; it has been widely used for controlling uncertain nonlinear systems due to its robustness and simplicity. However, only the matched uncertainty and disturbance can be rejected by the sliding mode controller. For mismatched uncertainty, the controller can be effective only under certain conditions or by combination with other methods, but generally it fails. Also, there is no methodical way of defining the Lyapunov function for the SMC. The backstepping approach is a nonlinear technique widely used in control design. The multiple advantages of this approach include its large set of globally and asymptotically stabilizing control laws and its capability to improve robustness and solve adaptive problems. This method uses a recursive procedure to link a selected Lyapunov function with a controller design and can suppress and synchronize nonlinear systems [9]. The scheme in this paper allows the controlled system to be robust to external disturbances and incorporating backstepping design processes to allow the designer to easily and systematically implement the controller. In order to utilize the benefits offered by both the sliding mode and the backstepping controller, these two have been combined to develop backstepping sliding mode controller [12] which will be robust to matched and mismatched uncertainties.The most control techniques of nonlinear system are based on the precise knowledge of the mathematical model, this latter is not often possible, because we can be confronted with inaccuracies due to uncertainties related to the studied processor to neglected dynamics.To maintain the same performances in the presence of major structural variations, the use of the fuzzy adaptive control is necessary. As in classical adaptive control, we can distinguish two cases: direct and indirect.In this paper, we propose a novel adaptive backstepping sliding mode control with fuzzy approximation strategy for the tracking control of unknown nonlinear system "Inverted Pendulum". First, an appropriate sliding mode surface is constructed, and it provides sufficient flexibility to shape the response of position tracking error, then the ABSMC scheme is proposed. To obtain a better perturbation rejection property, adaptive law is employed to compensate lumped perturbations. Thus it relaxes the requirement of the bound of lumped perturbation. The unknown functions of the nonlinear system are approximated by fuzzy systems, basing on the universal approximation theorem, where the parameters of fuzzy systems are adjusted using adaptation laws, based on the Lyapunov synthesis in order to ensure the global stability of the system and the convergence to zero of the tracking error. To solve the problem of chattering, which is a major disadvantage of the sliding mode technique; we approximate the discontinuous control by an adaptive fuzzy system.

Our contribution consist to do the combination of the techniques cited above, where we have chosen the sliding surfaces as the backstepping variables, this contribution based on the fuzzy approximation of the unknown system dynamics and also the switching control in order to minimize the chattering effect.

This paper is organized as follows: Section II gives a problem formulation of a Cart-Pendulum dynamics. In the section III, we present the synthesis of the proposed FABSM control, at the last part; the performances of the proposed method are shown using numerical simulation. Finally Section IV concludes this paper.

II. PROBLEME FORMULATION

A. Presentattion of the system dynamics

The Cart-Pendulum is a nonlinear system with two degree of freedom, is a basic model for the stability study of nonlinear systems. Fig.1 shows the Cart-Pendulum system whose dynamic model is presented as follows:[19]

$$\begin{cases}(m_c+m)\ddot{x}+ml\ddot{\theta}\cos\theta-ml\dot{\theta}^2 sin\theta=F\\ \ddot{x}cos\theta+l\ddot{\theta}-gsin\theta=0\end{cases}\tag{1}$$

Where: $\theta,\dot{\theta}\ and\ \ddot{\theta}$ represent the position, velocity and acceleration of the pendulum. $x,\dot{x}\ and\ \ddot{x}$represent the position, velocity and acceleration of the cart. F is the force acting on the cart. $m\ and\ m_c$are the masses respectively of the pendulum and the cart. $g$is the gravity. l is the half of length of the pendulum.

In order to simplify the state space model, we take as input the angular acceleration of the pendulum (rather than the force).

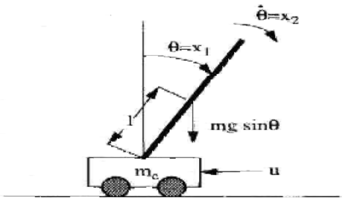

Fig. 1. The inverted pendulum system

The state space model of the Cart-Pendulum is given as follows:

$$\begin{cases}\dot{x}_1=x_2\\ \dot{x}_2=f(x)+g(x)u+d(t)\end{cases}\tag{2}$$

$$y(t)=x_1\tag{3}$$

Where: u and y are respectively, the input and the output of the system.$f(x)$and$g(x)$ are nonlinear unknown continuous smooth functions, such that:

$$f(x)=\frac{gsin(x_1)-\frac{mlx_2^2cos(x_1)\sin(x_1)}{m_c+m}}{l(\frac{4}{3}-\frac{mcos^2(x_1)}{m\_c+m})},\ g(x)=\frac{\frac{cos(x_1)}{m_c+m}}{l(\frac{4}{3}-\frac{mcos^2(x_1)}{m\_c+m})}$$

$d(t)$is the unknown bounded external disturbance. We assume just the upper limit of the perturbation, as $|d(t)|\le D$.

The state space variables are the position and the velocity of the pendulum $[x_1x_2]=[\theta\ \dot{\theta}]$ and the output is $y(t)$.

We assume that the system is always controllable, so $g^{-1}(x)$ exists and does not equal to zero.

The control objective,of this paper is to design a fuzzy adaptive backstepping sliding mode controller, such that the system output $y(t)$ follows the reference signal $x_d(t)$, under the constraint that all signal, must be bounded and the system be stable.

B. Fuzzy approximation

In this paper we construct the fuzzy logic system, with the following If-Then rules:

$$R_i: If\ x_1\ is\ F_1^i and\ ...\ and x_n\ is\ F_n^i\ then\ \psi\ is\ B^i,\quad i=1,2,..,n$$

The fuzzy logic system with the singleton fuzzifier, product inference and center average defuzzifier, are expressed in the following form: $\psi(x)=\frac{\sum_{i=1}^n\theta_i\prod_{j=1}^n\mu_{F_j^i}(x_j)}{\sum_{i=1}^n\left[\prod_{j=1}^n\mu_{F_j^i}(x_j)\right]}$

Where $x=[x_1,\dots,x_n]^T\in R^n,\mu_{F_j^i}(x_j)$ isthe membership of F_j^i, $\theta_i=\max_{y\in R}\mu_{B^i}(y)$, let:$\xi_i(x)=\frac{\sum_{i=1}^n\theta_i\prod_{j=1}^n\mu_{F_j^i}(x_j)}{\sum_{i=1}^n\left[\prod_{j=1}^n\mu_{F_j^i}(x_j)\right]}$

$\xi(x)=[\xi_1(x),\xi_2(x),\dots,\xi_n(x)]^Tand\theta=[\theta_1,\theta_2,\dots,\theta_n]^T$

Then the fuzzy logic system can be rewritten as follows:

$$\psi(x)=\theta^T\xi(x)$$

The following Lemma1, points out that the above fuzzy logic systems are capable to uniformly approximating any continuous nonlinear function, over a compact setΩ_x.

Lemma1:[1],[3],[8]

For any given continuous function$f(x)$ on a compact set$\Omega_x\subset R^n$; there exists a fuzzy logic system $\psi(x)$ In the form (4), such that for any given positive constant ε.

$sup_{x\in\Omega_x}|f(x)-\psi(x)|\le\varepsilon.$

Then the fuzzy system in the form (4) is a universal approximation, which can approximate the unknown nonlinear function $f(x)$ and $g(x)$ is modeled by fuzzy system $\hat{f}(x)$ and$\hat{g}(x)$ respectively. Then we have the following equations:

$$\begin{cases}f(x)=\hat{f}(x)+\Delta f(x)\\ g(x)=\hat{g}(x)+\Delta g(x)\end{cases}\begin{cases}\hat{f}(x)=\hat{\theta}_f^T\xi_f(x)\\ \hat{g}(x)=\hat{\theta}_g^T\xi_g(x)\end{cases}$$

Such that: $w=\ \Delta f(x)+\Delta g(x)u$, is the approximation error

III. DESIGN OF ADAPTIVE BACKSTEPPING SLIDING MODE LAW

The recursive nature of the propose control design is similar to the standard backstepping methodology. However the proposed control design uses backstepping to design controllers with a zero order sliding surface at each step [18]. The benefit of this approach is that each actual controller can compensate the unknown bounded terms$d(t)$, the design proceeds as follows:
For the first step we consider zero-order sliding surface:

$$s_1=x_1-x_{1d}\tag{4}$$

Let the first Lyapunov function candidate:

$$V_1(s)=(\tfrac{1}{2})s_1^2\tag{5}$$

The time derivation of (5) is given by:

$$\dot{V}_1(s) = s_1\dot{s}_1 = s_1(x_2 - \dot{x}_{1d}) = -c_1 s_1^2 + s_1 s_2 \quad (6)$$

The stabilization of $s_1$can be obtained by introducing a new virtual control x_{2d}, such that:

$$x_{2d} = \dot{x}_{1d} - c_1 s_1 , \qquad c_1 > 0 \quad (7)$$

Where $c_1$is the feedback gain, such that x_{2d} has been chosen in order to eliminate the non linearity and getting $\dot{V}_1(s) < 0$. The term $s_1 s_2$de $\dot{V}_1(s)$will be eliminated in the next step, so the first sub system is stabilized.

For the second step we consider the following zero-order sliding surface:

$$s_2 = x_2 - x_{2d} = x_2 - \dot{x}_{1d} + c_1 s_1 \quad (8)$$

The augmented Lyapunov function is given by:

$$V_2(s_1, s_2) = V_1 + \left(\frac{1}{2}\right) s_2^2 + \left(\frac{1}{2\mu_f}\right) \tilde{\theta}_f^T \tilde{\theta}_f + \left(\frac{1}{2\mu_g}\right) \tilde{\theta}_g^T \tilde{\theta}_g \quad (9)$$

With:$\tilde{\theta}_f = \theta_f - \hat{\theta}_fand\tilde{\theta}_g = \theta_g - \hat{\theta}_g$

$\theta_f$and$\theta_g$are the parameters vectors respectively of functions *f*(*x*)and *g*(*x*) , and $\hat{\theta}_f$, $\hat{\theta}_g$, are theirs estimations.

The time derivative of $V_2(s_1, s_2)$ is then:

$$\dot{V}_2(s_1, s_2) = s_1\dot{s}_1 + s_2\dot{s}_2 + \left(\frac{1}{\mu_f}\right) \tilde{\theta}_f \dot{\tilde{\theta}}_f + \left(\frac{1}{\mu_g}\right) \tilde{\theta}_g \dot{\tilde{\theta}}_g \quad (10)$$

We have:$\dot{\tilde{\theta}}_f = -\dot{\hat{\theta}}_f$ and $\dot{\tilde{\theta}}_g = -\dot{\hat{\theta}}_g$

Then:

$$\dot{V}_2(s_1, s_2) = -c_1 s_1^2 + s_2\left[s_1 + \hat{\theta}_f^T \xi_f(x) + \hat{\theta}_g^T \xi_g(x) u - \dot{x}_{2d} + w + d(t)\right] + \tilde{\theta}_f^T \left[s_2 \xi_f(x) - \left(\frac{1}{\mu_f}\right) \dot{\hat{\theta}}_f\right] + \tilde{\theta}_g^T \left[s_2 \xi_g(x) u - \left(\frac{1}{\mu_g}\right) \dot{\hat{\theta}}_g\right] \quad (11)$$

With:

$$\dot{x}_{2d} = -c_1 s_2 + c_1^2 s_1 + \ddot{x}_{1d} \quad (12)$$

The negativity of the Lyapunov function, allows getting the following control law:

$$u = u_{eq} - \frac{1}{\hat{g}} u_{sw} \quad (13)$$

$$u_{sw} = -k sign(s_2) \quad (14)$$

$u_{sw}$is the switching control.

So:

$$\dot{V}_2(s_1, s_2) = -c_1 s_1^2 + s_2 \left[s_1 + \hat{\theta}_f^T \xi_f(x) + \hat{\theta}_g^T \xi_g(x) \left(u_{eq} - \frac{1}{\hat{\theta}_g^T \xi_g(x)} u_{sw}\right) + c_1 s_2 - c_1^2 s_1 - \ddot{x}_{1d} + d(t) + w\right] + \tilde{\theta}_f \left[s_2 \xi_f(x) - \left(\frac{1}{\mu_f}\right) \dot{\hat{\theta}}_f\right] + \tilde{\theta}_g \left[s_2 \xi_g(x) u - \left(\frac{1}{\mu_g}\right) \dot{\hat{\theta}}_g\right] \quad (15)$$

w is the approximation errors of functions *f*(*x*) and *g*(*x*).

The equivalent control is then:

$$u_{eq} = \frac{1}{\hat{\theta}_g^T \xi_g(x)} \left[-c_2 s_2 - s_1 - \hat{\theta}_f^T \xi_f(x) + \ddot{x}_{1d} - c_1 s_2 + c_1^2 s_1\right] \quad (16)$$

So the control law becomes:

$$u = \frac{1}{\hat{\theta}_g^T \xi_g(x)} \left[-c_2 s_2 - s_1 - \hat{\theta}_f^T \xi_f(x) + \ddot{x}_{1d} - c_1 s_2 + c_1^2 s_1 - u_{sw}\right] \quad (17)$$

Choosing the adaptive laws as follows:

$$\begin{cases} \dot{\hat{\theta}}_f = \mu_f s_2(x) \xi_f \\ \dot{\hat{\theta}}_g = \mu_g s_2(x) \xi_g u \end{cases} \quad (18)$$

The equation (15) is developed to:

$$\dot{V}_2 = -c_1 s_1^2 - c_2 s_2^2 - s_2(k sign(s_2) - d(t) - w) \quad (19)$$

Introducing the norm, we get:

$$\dot{V}_2 \leq -c_1 s_1^2 - c_2 s_2^2 - |s_2|(k - \gamma) < 0 \quad (20)$$

We have: $|d(t) + w| \leq \gamma$

$\{c_2, k\}$are positive constants, with $k > \gamma$.

$sign(.)$is the usual sign function.

This proves the decreasing of Lyapunov function, which means that the equilibrium $x_1 = x_{1d}$ is globaly asymptotically stable and $\lim_{t\to\infty} x_1(t) = x_{1d}$. Thereforewe could ensure the stability of the closed loop system.

Indeed, the value of the constant*k* depends on the upper bound of the structural uncertainties and external disturbances, which are unknown. In order to resolve this problem, we propose in the following to modify the previous control law, using a fuzzy adaptive system $\hat{h}(s)$ [11], having the sliding surface as input, to approximate the term *k.sign(s(x))*. Thus the fuzzy nature of this latter allows eliminating perfectly the phenomenon of chattering, while its adaptive aspect is designed to best approximate the constant, and therefore enfranchise a priori knowledge about the upper bounds of the structural uncertainties and external perturbations.

The derivative of the sliding surface given in (8), is as follows:

$$\dot{s}_2(x,t) = \theta_f^T \xi_f(x) + \theta_g^T \xi_g(x) u + \theta_h^T \xi_h(s) + w' - \hat{h}^*(s) + d(t) - \dot{x}_{2d} \quad (21)$$

Ou : $\hat{h}(s) = \hat{\theta}_h^T \xi_h(x)$ and $w' = \Delta f(x) + \Delta g(x) u - \Delta h(x)$

We consider the following Lyapunov function:

$$V_2(s_1, s_2) = V_1 + \frac{1}{2} s_2^2(x) + \left(\frac{1}{2\mu_f}\right) \tilde{\theta}_f^T \tilde{\theta}_f + \left(\frac{1}{2\mu_g}\right) \tilde{\theta}_g^T \tilde{\theta}_g + \left(\frac{1}{2\mu_h}\right) \tilde{\theta}_h^T \tilde{\theta}_h \quad (22)$$

The derivative of this latter introducing the control law (13), is given by:

$$\dot{V}_2 = -c_1 s_1^2 + s_2\left[s_1 - c_1^2 s_1 + c_1 s_2 - \ddot{x}_{1d} + \hat{\theta}_f^T \xi_f(x) + \hat{\theta}_g^T \xi_g(x) u_{eq} - u_{sw} + w - \hat{h}^*(s) + d(t)\right] +$$

$$\tilde{\theta}_f^T\left[s_2\xi_f(x)-\left(\frac{1}{\mu_f}\right)\dot{\hat{\theta}}_f\right]+\tilde{\theta}_g^T\left[s_2\xi_g(x)u-\left(\frac{1}{\mu_g}\right)\dot{\hat{\theta}}_g\right]+$$
$$\tilde{\theta}_h^T\left[s_2\xi_h(s)-\left(\frac{1}{\mu_h}\right)\dot{\hat{\theta}}_h\right] \quad (23)$$

Where: $u=u_{eq}-\frac{1}{\hat{\theta}_g^T\xi_g(x)}\hat{\theta}_h^T\xi_h(s)andu_{sw}=\hat{\theta}_h^T\xi_h(s)$.

Consequently the equivalent control law is given by:

$$u_{eq}=\frac{1}{\hat{\theta}_g^T\xi_g(x)}[-c_2s_2-s_1-\hat{\theta}_f^T\xi_f(x)+\hat{\theta}_h\xi_h(s)+\ddot{x}_{1d}+c_1(c_1s_1-s_2)] \quad (24)$$

To ensure the negativity of the Lyapunov function derivative, we choice the following control law:

$$u=\frac{1}{\hat{\theta}_g\xi_g(x)}[-c_2s_2-s_1-\hat{\theta}_f\xi_f(x)+\hat{\theta}_h\xi_h(s)+\ddot{x}_{1d}+c_1(c_1s_1-s_2)-u_{sw}] \quad (25)$$

The adaptation laws are given as follows:

$$\begin{cases}\dot{\hat{\theta}}_f=\mu_f s_2(x)\xi_f(x)\\ \dot{\hat{\theta}}_g=\mu_g s_2(x)\xi_g(x)u\\ \dot{\hat{\theta}}_h=\mu_h s_2(x)\xi_h(s)\end{cases} \quad (26)$$

The optimal value of $\hat{h}(s)$is such that:

$$|\hat{h}^*(s)|\geq|w'|+|d|$$

From equation (23), we have:

$$\dot{V}_2\leq -c_1s_1^2-c_2s_2^2+|s_2|\left(|w'|+|d|-|\hat{h}^*(s)|\right)<0 \quad (27)$$

Which implies that: $\dot{V}_2\leq 0$, so the closed loop system is stable and robust.

The block scheme of the control strategy is given by the Fig.2

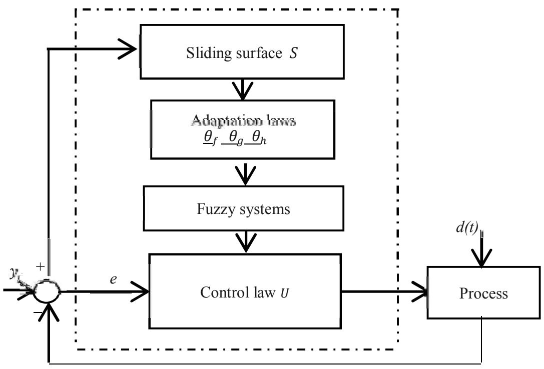

Fig. 2. Control scheme

IV. Simulation Results

In order to verify the performance and robustness of the proposed control law, when applied to the inverted pendulum, these simulations were made considering different cases and conditions. We show the results, first when applying only the adaptive backstepping control law, secondly when applying the adaptive backstepping sliding mode control law, introducing a perturbation, and finally when using the fuzzy adaptive backstepping sliding mode control law. The structure of the inverted pendulum is shown in Fig.1.

Where $m_c=1kg, m=0.1kg, and\ l=0.5m$, the reference signal is given by: $x_d(t)=\frac{\pi}{60}\sin(t)$, and the initial conditions are: $x_1(0)=\frac{\pi}{60}, x_2(0)=0$.

Considering the following membership functions for $f(x)$and $g(x)$[19]

$\mu_{F_i^1}(x_i)=\exp(-(x_i+\frac{\pi}{6}/\frac{\pi}{24})^2), \mu_{F_i^3}(x_i)=\exp(-(x_i/\frac{\pi}{24})^2)$,

$\mu_{F_i^2}(x_i)=\exp\left(-\left(x_i+\frac{\frac{\pi}{12}}{\frac{\pi}{24}}\right)^2\right), \mu_{F_i^4}(x_i)=\exp(-(x_i-\frac{\pi}{12}/\frac{\pi}{24})^2)$,

$\mu_{F_i^5}(x_i)=\exp(-(x_i-\frac{\pi}{6}/\frac{\pi}{24})^2)$;

In order to construct the fuzzy system for the signal$h(S)$, which approximate the switching control and eliminate the chattering phenomenon, we divide the discourse universe (the surface$S$) on three sets: « Positive », « Zero » and « negative » to which are associated the following membership functions:

$$\mu_{positive}(S)=1/\left(1+8.exp(S-0.1)\right)$$
$$\mu_{Zero}(S)=1/-(S/0.5)^2$$
$$\mu_{negative}(S)=1/\left(1-8.exp(S-0.1)\right)$$

Three fuzzy rules are used to deduce the signal:

$$R^1: if S is Negative then \hat{h}(S)=-C$$
$$R^2: if S is Zero then \hat{h}(S)=0$$
$$R^3: if S is Positive then \hat{h}(S)=C$$

1) Results of the adaptive backstepping control

The results of the adaptive backstepping strategy without perturbation,are shown in Fig.3, Fig.4 and Fig.5.

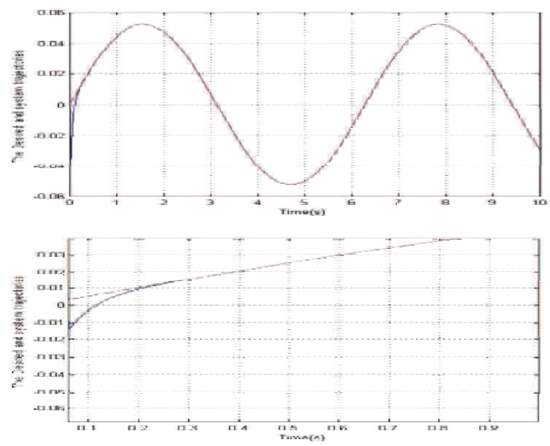

Fig. 3. Pendulum angle tracking

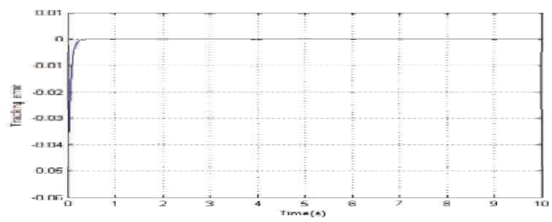

Fig. 4. Tracking error

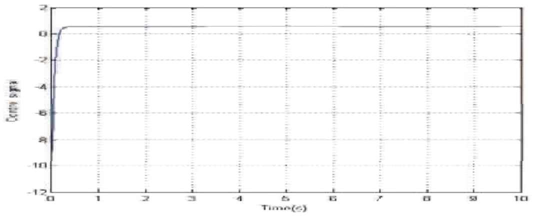

Fig. 5. Control signal

The simulation results for the adaptive backstepping with perturbation (parametric uncertainty in the masses and external perturbation)are in Fig.6, Fig.7 and Fig.8.

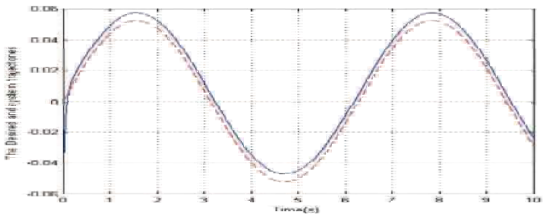

Fig.6. Pendulum angle tracking

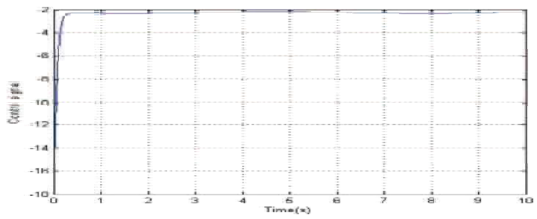

Fig.7.Tracking error

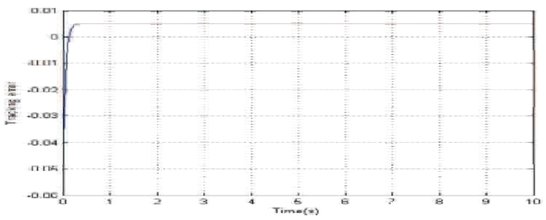

Fig.8. Control signal

From the results of the backstepping technique, we can see that this later gives good performances, either in response time (0.25s) or in tracking error. But this technique loses its performance's characteristic in the presence of perturbations.We can see a significant tracking error. In this case we have thought to introduce the sliding mode control in order to guaranty the robustness of the controller. The results of the combination backstepping and sliding mode are given in the next section.

2) Results of the adaptivebackstepping sliding mode control

The results of the adaptive backstepping sliding mode control with presence of perturbations are shown in Fig.9, Fig.10, Fig.11.

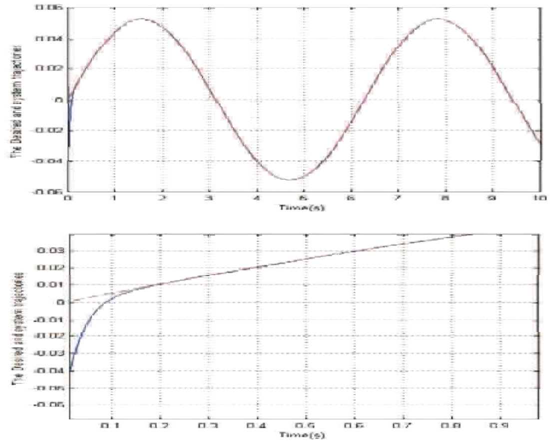

Fig. 9. Pendulum angle tracking

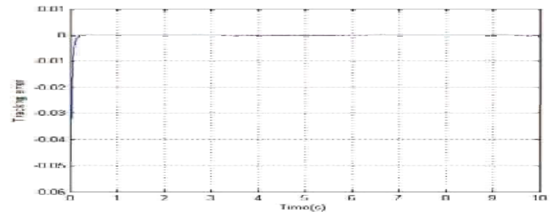

Fig. 10. Tracking error

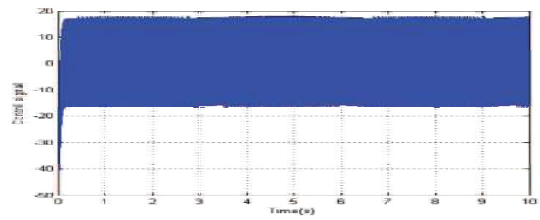

Fig. 11. Control signal

From the figures Fig.9, Fig.10 and Fig.11 which represent the results of the combination backstepping and sliding mode, we can conclude an improvement in the response time of the system (1.8s) compared to the response time in the adaptive backstepping technique(2s), while keeping the convergence to zero of the error, that means this combination of the two techniques is more robust than the first one, despite that, the backstepping sliding mode control has two problems, firstly the chattering phenomena associated to the sliding mode control, which presents a major drawback, because it can excite the dynamic of the commutation in high frequency, and secondly we can't handle the control when we have uncertainties in the system dynamics is. In order to overcome these problems, we introduce the fuzzy control, where wehave used the fuzzy systems to approximate the unknown functions and the switching control. The results are presented in the next section.

3) Results of the fuzzy adaptive backstepping sliding mode control

The results of the fuzzy adaptive backstepping sliding mode control are shown in Fig.12, Fig.13, Fig.14.

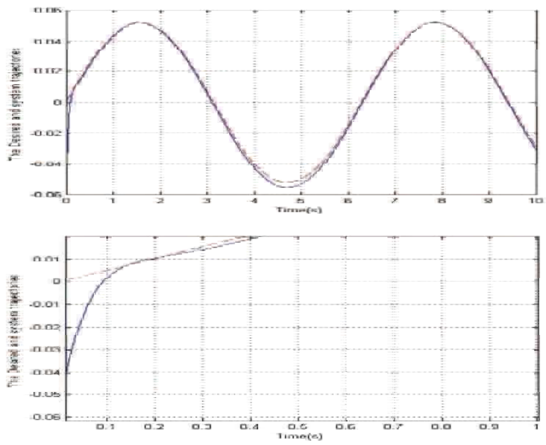

Fig. 12. Pendulum angle tracking

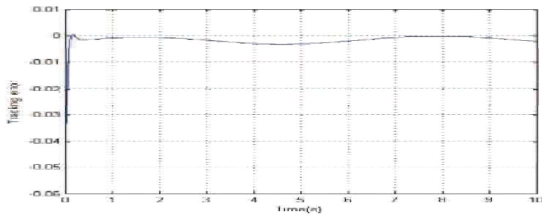

Fig. 13. Tracking error

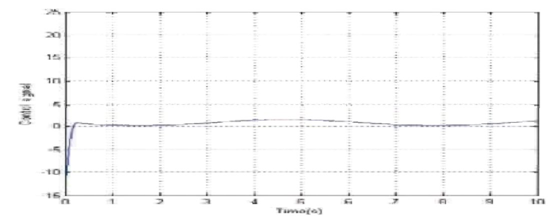

Fig. 14. Control signal

From figures Fig. 12, Fig .13 and Fig. 14, we can see clearly that the simulation results of the approximated model of the inverted pendulum show that even the dynamics are unknown and with presence of perturbations, the response of the system tracks its reference model with a very small error (0.01) and with a good response time. The chattering phenomenon matched in the sliding mode control is eliminated in those results, using the approximation of the switching control by fuzzy systems.Those results demonstrate the efficiency and the robustness of proposed approach, against disturbances and parameter variations.

V. CONCLUSION

In this study, a fuzzy adaptive backstepping sliding mode control algorithm for the nonlinear system has been developed, which integrate a fuzzy adaptive backstepping methodology and the sliding mode control strategy. The combined strategies are shown to have the advantages of the sliding mode and the backstepping approach. The adaptive sliding mode backstepping control is proposed on the basis of Lyapunov stability criteria, which can be applied to uncertainties and disturbances inputs.

The objective is achieved, the output signal tracks the desired reference model, with a best response time (≈0.15s) compared to the obtained results in other works, and the fuzzy logic control could be applied to approximate the nonlinear unknown dynamic of the system and to reduce the chattering phenomena appears in the backstepping sliding mode control.

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