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Invited Review Paper

Polytopic LPV approaches for intelligent automotive systems: State of the art and future challenges

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ABSTRACT

With more and more stringent requirements on driving comfort, safety and fuel economy, polytopic LPV approaches have become popular in intelligent automotive control systems due to their merits in dealing with the complex nonlinearities. This survey starts with a review on control theory of polytopic LPV systems. Stability analysis and controller design are provided with techniques in obtaining less conservative results. Then, some key applications in vehicle dynamics control are provided. Several LPV models concerning the vertical dynamics, lateral dynamics and integrated dynamics are introduced. Different polytopic LPV control designs are summarized taking various settings on time-varying parameters into account. Moreover, polytopic LPV approaches in vehicle path following and powertrain control are concluded, including the applications in internal combustion engines, electric vehicles and aftertreatment systems. Finally, from recent advances on polytopic LPV control theory and automotive applications, future research directions and related challenges are discussed.
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1. Introduction

Linear-parameter-varying (LPV) technique has been demonstrated as an effective approach to deal with complex nonlinear systems [1]. Depending on the type of trajectories of the parameters, the LPV framework can be used to represent various classes of nonlinear systems, including LPV systems with slowly or arbitrarily fast varying parameters [2], switched systems [3], hybrid dynamical systems [4], and periodic systems [5]. The main interest of LPV approaches is to make possible the extension of some linear concepts to the case of nonlinear systems such as $\mathcal{H}_\infty$ control, sensitivity shaping, $\mathcal{D}$-stability, etc. Consequently, LPV approaches have been exploited for modeling, estimation and control in a variety of engineering applications [1,2,6], especially automotive nonlinear systems [7]. One of the major challenges of LPV approaches arises from deriving less conservative optimization-based solutions with affordable computational load for complex high-dimensional nonlinear systems.

With the rapid development of modern automotive industry, increasingly stringent requirements on passenger comfort, fuel economy, vehicle safety, and pollutant emissions have been proposed. The intelligent automotive systems (IASs) emerge to copy with those various requirements, of which a major task is the effective control and estimation algorithm design. Equipped with an increasing number of sensors, actuators and other integrated devices, modern IASs involve more and more nonlinearities and uncertainties. Although kinds of new technologies have been adopted in modern vehicles [8], the complex nonlinearities and uncertainties in IASs give rise to great challenges in the controller and estimator synthesis. Due to the legacy control system and the highly nonlinear and highly complex vehicle systems, most vehicle control features are still heavily relying on feedforward proportional-integral-derivative (PID) based controls with gain scheduling as a way to address the nonlinearity. In addition, most of the controller loops are designed for single-input-single-output (SISO) which leads to significant optimality deficiency when SISO loops are highly coupled. The usual way to deal with these interactions is to de-tune individual control loop performances. Despite a solid theoretical foundation, LPV methodology is still not widely used

in industrial applications, especially in automotive industry. Therefore, it is timely to conduct a review on the use of LPV approaches for IAS applications to show both the interests and the recent advances which have been achieved in this topic.

This paper first provides in Section 2 a concise overview on LPV approaches. The goal is not to comprehensively review the literature which already includes several monographs [1,9,2,10,7], surveys [6,11,12], and numerous technical articles therein. Instead, a selective list of notable references is given to describe LPV approaches, especially the mainstreams to reduce the related design conservatism. Our goal is to provide the insights of LPV technique which will be useful for the control design of real-world applications. Moreover, the presented methods to reduce the design conservatism can be exploited as practical guidelines to improve the control performance for complex engineering systems. Among various existing LPV approaches [2], *e.g.*, polytopic LPV design, linear fractional transformation (LFT) LPV design, gridding-based LPV design, here the emphasis is put on polytopic LPV framework without loss of generality. Note that systems with other parameter dependencies can be transformed into polytopic LPV models [9]. Moreover, polytopic LPV approaches have become very popular, at least from the viewpoint of the number of related publications [6]. In Section 3, we discuss the polytopic LPV approach in vehicle dynamics control. Vehicle dynamic systems can be modeled as polytopic LPV systems because of the time-varying parameters. Several LPV models concerning vehicle vertical dynamics, lateral dynamics and integrated dynamics are briefly introduced. According to different time-varying parameters considered, various gain-scheduled controllers to improve the driving comfort and safety are concluded. Since some parameters vary independently, methods on shrinking the polytopic to obtain less conservative results are provided as well. Autonomous vehicles have been regarded as a promising future for complex IASs. The successful application of polytopic LPV to the control of autonomous vehicles is reviewed in Section 4. Besides the autonomous vehicles, the powertrain control has gained considerable attentions as well during the past decades. The polytopic LPV approaches for intelligent powertrain systems are reviewed in Section 5. In Section 6, we present our viewpoint on the current research trends and challenges. This paper aims to complement other surveys on LPV theory [11,12,1] and applications [6] with a special focus on polytopic LPV approaches for IASs [13,14]. Note that recent comprehensive surveys on some mainstream techniques for path following control of autonomous vehicles, including pure pursuit, feedback linearization, MPC control, and many others, can be also found in [15,16].

Notation $\mathcal{I}_N$ denotes the set $\{1,2,\ldots,N\}$. $\mathbb{R}$ is the field of real numbers. For a vector $x \in \mathbb{R}^n$ and $i \in \mathcal{I}_n$, x_i denotes the ith entry of x, and $\|x\|_\infty = \max_{i=1,\ldots,n} x_i$ denotes the infinity norm of x. I denotes the identity matrix of appropriate dimension. For a matrix X, $X^\top$ indicates its transpose. For any square matrix X, $X \succ 0$ indicates a symmetric positive-definite matrix, and $\mathrm{He}X = X + X^\top$. $\mathrm{diag}(X_1, X_2)$ denotes a block-diagonal matrix composed of X_1, X_2. The symbol $\star$ represents matrix blocks that can be deduced by symmetry. The time dependency of the variables is omitted when convenient.

2. Polytopic linear parameter-varying systems

This section first provides an automotive application to motivate the need of LPV control in improving the practical performance of IASs. Then, after a brief description, we discuss some key points related to the stability analysis and synthesis of polytopic LPV systems.

2.1. A motivating automotive application

Adaptive cruise control (ACC) is an advanced driver-assistance system for road vehicles that automatically adjusts the vehicle speed to maintain a safe distance from vehicles ahead [14], see Fig. 1. Due to its important role for the safety issue of vehicles in the same lane, these ACC systems have been the research and development focus of the automotive industry [17]. Hereafter, we consider the multi-objective control problem of an ACC system to motivate the the need for LPV control in IAS applications.

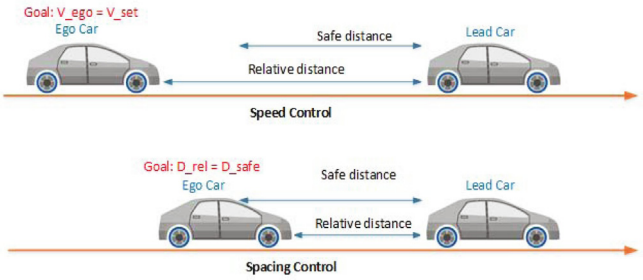


Fig. 1. Illustration of vehicle ACC systems [18]. VegoV_- ego is the ego vehicle velocity, V_- set is the ego vehicle velocity setpoint, D_- rel is the distance relative between lead vehicle and ego vehicle, and D_- safe is the safe distance between lead vehicle and ego vehicle.

The longitudinal dynamics of autonomous vehicles can be described by the following differential equations [19]:

$$\begin{aligned} M_v\dot{V}_x &= F_l - F_d, \\ \tau\dot{F}_l &= -F_l + u, \end{aligned} \tag{1}$$

where V_x is the longitudinal speed, F_l is the longitudinal force realized on the wheels, M_v is the total mass of the vehicle, τ denotes the time constant of the longitudinal actuators. The control input u represents the required longitudinal control force. The longitudinal disturbance force F_d aims at taking into account the aerodynamics and the road conditions as [20]

$$F_d = C_a V_x^2 + C_r M_v g\cos\phi + M_v g\sin\phi, \tag{2}$$

where ϕ is the road slope, C_a and C_r are the vehicle parameters related to the aerodynamics and rolling resistances. From (1) and (2), the longitudinal dynamics can be represented in the following state-space form:

$$\dot{x} = A(V_x)x + Bu + Ew(\phi), \tag{3}$$

where $x = [\,V_x \quad F_l\,]^\top$ is the state vector, $w(\phi) = C_r\cos\phi + \sin\phi$ is the system disturbance, and

$$A(V_x) = \begin{bmatrix} -\frac{C_a}{M_v}V_x & \frac{1}{M_v} \\ 0 & -\frac{1}{\tau} \end{bmatrix},\ B = \begin{bmatrix} 0 \\ \frac{1}{\tau} \end{bmatrix},\ E = \begin{bmatrix} M_v g \\ 0 \end{bmatrix}. \tag{4}$$

Remark that the dynamic matrix $A(V_x)$ of system (3) *explicitly* depends on the vehicle speed, which is time-varying and bounded as follows [14]:

$$V_x \in [5, 30]\mathrm{m/s},\ \dot{V}_x \in [-3.5, 2.5]\mathrm{m/s}^2. \tag{5}$$

Note that classical linear time-invariant approaches, *e.g.*, linear quadratic regulator (LQR), LQR with preview of the disturbance, model predictive control, loop-shaping robust control, etc., can be applied to the LPV system (3) for related control problems. A notable survey on these linear control approaches for vehicle dynamics applications can be found in [13]. Moreover, MPC technique has shown large potential for use in automotive applications [21]. However, when the system to be dealt with is highly nonlinear and uncertain, linear MPC technique may not provide a satisfactory control performance. Robust and/or nonlinear MPC control schemes should be required in this situation. Unfortunately, there are still some fundamental challenges to overcome when using robust/nonlinear MPC techniques [13,21]. First, the calibration effort for MPC controllers can be costly in many cases. Second, the control design of nonlinear MPC controllers are still too computationally complex, and low-complexity explicit control laws or fast optimization algorithms are necessary. Third, guaranteeing the stability of MPC a priori, without increasing excessively the algorithm complexity is still widely open. Finally, several applications have significant nonlinearities and uncertainties, as the usual case of intelligent automotive systems, which complicate MPC design and implementation. LPV control technique could be an effective alternative to overcome these major drawbacks for such complex nonlinear systems. In particular, taking into account the information on the time-varying parameters such as in (5), the LPV design conservatism can be significantly reduced to further improve the practical control performance [1].

Hereafter, we review some basic features of LPV control to highlight its ability to take into account the *time-varying* nature of system parameters in the design procedure for performance improvements. The mainstream of current research on LPV control theory, with a focus on polytopic approaches, are also discussed. These discussions aim at providing different theoretical directions that could be performed to further improve the practical performance of real-world LPV control systems such as in the case of IAS applications.

2.2. Polytopic LPV system description

For generality, we consider the LPV system (3) in its general state-space form as

$$\begin{aligned} \dot{x} &= A(\theta)x + B(\theta)u + E(\theta)w, \\ y &= C(\theta)x + F(\theta)w, \end{aligned} \tag{6}$$

where $x \in \mathbb{R}^{n_x}$ is the state of the system, $u \in \mathbb{R}^{n_u}$ is the control input, $w \in \mathbb{R}^{n_w}$ is the disturbance input, $y \in \mathbb{R}^{n_y}$ is the system output, and $\theta \in \mathbb{R}^p$ is the vector of unknown *time-varying* parameters whose measurement is available in real time for gain scheduling control. Assume that the parameter $\theta = [\,\theta_1 \ldots \theta_p\,]^\top$ and its unknown rate of variation $\dot{\theta}(t)$ are smooth and respectively valued in the hypercubes

$$\begin{aligned} \mathcal{S}_\theta &= \left\{ (\theta_1, \ldots, \theta_p)^\top : \theta_j \in [\underline{\theta}_j, \overline{\theta}_j],\ j \in \mathcal{I}_p \right\}, \\ \mathcal{S}_v &= \left\{ \left(\dot{\theta}_1, \ldots, \dot{\theta}_p\right)^\top : \dot{\theta}_j \in [\underline{v}_j, \overline{v}_j],\ j \in \mathcal{I}_p \right\}, \end{aligned} \tag{7}$$

where $\underline{\theta}_j \leqslant \overline{\theta}_j$ (respectively $\underline{\upsilon}_j \leqslant \overline{\upsilon}_j$) are *known* lower and upper bounds on θ_j (respectively $\dot{\theta}_j$), for $j \in \mathcal{I}_p$. We assume that the time-varying matrices $A(\theta)$, $B(\theta)$, $C(\theta)$, $E(\theta)$ and $F(\theta)$ of system (6) are continuous on the hypercube $\mathcal{S}_\theta$. Note that condition (7) of the time-varying parameter θ is a *generalized* version of (5).

Remark 1. Several methods have been proposed to obtain the LPV model in (6) from a nonlinear system, for instance Jacobian linearization [22], function substitution [23], state transformation [24]. More details on analytical LPV modeling and experimental LPV modeling can be found in [7,10], respectively. Note that for a given nonlinear system, its LPV representation is not unique and different models yield different properties with respect to stability analysis and/or control performance. Especially, an LPV model can suffer from *overbounding*, *i.e.*, the parameters are related to each other by inherent couplings, which can increase the numerical complexity and the conservatism of the design results [25]. The so-called *parameter set mapping* approach in [26] can be used to obtain a less conservative LPV representation.

Using the sector nonlinearity approach [27, Ch. 2], the LPV model (6) can be *equivalently* rewritten in the polytopic form

$$\begin{aligned} \dot{x} &= \sum_{i=1}^{N} \eta_i(\theta)(A_i x + B_i u + E_i w), \\ y &= \sum_{i=1}^{N} \eta_i(\theta)(C_i x + F_i w), \end{aligned} \tag{8}$$

with $N = 2^p$. The constant matrices of appropriate dimensions A_i, B_i, C_i, E_i, and F_i, for $i \in \mathcal{I}_r$, represent the set of N local linear sub-models, which are defined as

$$\Pi_i = \Pi(\theta)|_{\eta_i(\theta)=1}, \quad \text{for} \quad \forall \Pi \in \{A, B, C, E, F\}.$$

The weighting functions $\eta_i(\theta)$, for $i \in \mathcal{I}_N$, are continuously differentiable and belong to the simplex, defined as

$$\Xi_\theta = \left\{ \eta(\theta) \in \mathbb{R}^N : \sum_{i=1}^{N} \eta_i(\theta) = 1,\ \eta_i(\theta) \geq 0,\ \forall \theta \in \mathcal{S}_\theta \right\}.$$

Since $\left(\theta, \dot{\theta}\right) \in \mathcal{S}_\theta \times \mathcal{S}_\upsilon$, with $\mathcal{S}_\theta$ and $\mathcal{S}_\upsilon$ defined in (7), then the lower bound ϕ_{i1} and the upper bound ϕ_{i2} of $\dot{\eta}_i(\theta)$ can be easily computed from the analytical expression of $\eta_i(\theta)$ as

$$\dot{\eta}_i(\theta) \in [\, \phi_{i1}, \quad \phi_{i2} \,], \quad \phi_{i1} \leqslant \phi_{i2}, \quad i \in \mathcal{I}_N. \tag{9}$$

Remark 2. The sector nonlinearity approach [27] allows deriving an *exact* polytopic form of the generic LPV system (6). The weighting functions $\eta_i(\cdot)$, for $i \in \mathcal{I}_N$, can capture the parameter nonlinearities, *i.e.*, they can be a *nonlinear* function of components of θ. Hence, stability and control methods based on the polytopic LPV model (8) can be applied to a larger class of parametric dependencies than, *e.g.*, linear, affine, rational or linear fractional transformation (LFT).

2.3. Lyapunov-based stability of polytopic LPV systems

For stability analysis, we consider an autonomous polytopic LPV system (8) of the form

$$\dot{x} = \sum_{i=1}^{N} \eta_i(\theta) A_i x. \tag{10}$$

In the sequel, we provide an overview on Lyapunov-based stability analysis of the polytopic LPV system (10).

2.3.1. Quadratic stability

The quadratic stability analysis of system (10) is a direct extension of the result for linear time-invariant systems in [28]. Consider a quadratic Lyapunov function of the form

$$V(x) = x^\top P x, \quad P \succ 0. \tag{11}$$

The following quadratic stability result is readily obtained.

Theorem 1. *The equilibrium of the LPV system in (10) is globally asymptotically stable if there exists a common positive-definite matrix P such that*

$$A_i^\top P + P A_i \prec 0, \quad i \in \mathcal{I}_N. \tag{12}$$

Note that condition (12) is represented in the form of linear matrix inequalities (LMIs). Hence, the stability analysis can be effectively performed with available numerical solvers [28]. This particular feature has sparked a growing interest in LPV-based approaches for nonlinear control theory and its real-world applications. The insights on the state of the art of prominent results in the early of this century are discussed in [12,11], whereas more recent theoretical developments of LPV systems and control can be found in [10,2,1]. Reference [6] also provides an application-oriented survey on this research topic.

For stability analysis, a common Lyapunov matrix P is used to check the stability for all local linear subsystems. Moreover, the weighting functions of the LPV system (10) are not involved in the stability condition. For these reasons, quadratic stability analysis makes no difference between time-invariant parameters, slowly varying parameters and arbitrarily fast varying parameters. Despite its simplicity, this type of stability may lead to over-conservative results [2]. Therefore, a considerable research effort has been devoted to the conservatism reduction issue [29,1,30–32]. The current mainstream to reduce the conservatism of LPV model-based approaches is based on the choices of different families of Lyapunov functions. These choices can be accompanied with an S-variable approach [33] to introduce slack variables for relaxation purposes [34]. Some insights of this research mainstream are discussed hereafter.

2.3.2. Poly-quadratic stability

The conservatism of using a single quadratic Lyapunov function (11) for LPV stability analysis is illustrated in [2]. A natural way to overcome this major drawback consists in constructing Lyapunov functions that are explicitly parameter dependent. To illustrate the idea of using slack variables for relaxations purposes, consider the following poly-quadratic Lyapunov function:

$$V(x) = x^\top \mathcal{P}(\theta) x, \tag{13}$$

where $\mathcal{P}(\theta) = \sum_{i=1}^{N} \eta_i(\theta)(P_i + X)$, and $P_i + X \succ 0$, for $i \in \mathcal{I}_N$. The time-derivative of the Lyapunov function (13) is given by

$$\dot{V}(x) = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}^\top \begin{bmatrix} \dot{\mathcal{P}}(\theta) & \star \\ \mathcal{P}(\theta) & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix}.$$

Remark 3. Note that the quadratic Lyapunov function (11) is directly recovered from (13) with $P_1 = \ldots = P_N = P$ and $X = 0$. Then, function (11) is only a special case of the poly-quadratic Lyapunov function (13).

Since $\eta(\theta) \in \Xi_\theta$, it follows that $\sum_{i=1}^{N} \dot{\eta}_i(\theta) X = 0$, for any matrix X. Then, $\dot{\mathcal{P}}(\theta)$ can be rewritten in the following form [35]:

$$\dot{\mathcal{P}}(\theta) = \sum_{k=1}^{N-1} \dot{\eta}_k(\theta)(P_k + X - P_N) + \dot{\eta}_N(\theta) X = \sum_{k=1}^{N-1} \sum_{l=1}^{2} \left[\omega_{kl}(\theta)\phi_{kl}\mathcal{X} + \frac{1}{N-1}\omega_{Nl}(\theta)\phi_{Nl} X \right], \tag{14}$$

with $\mathcal{X} = P_k + X - P_N$, and

$$\omega_{k1}(\theta) = \frac{\phi_{k2} - \dot{\eta}_k(\theta)}{\phi_{k2} - \phi_{k1}}, \quad \omega_{k2}(\theta) = \frac{\dot{\eta}_k(\theta) - \phi_{k1}}{\phi_{k2} - \phi_{k1}}. \tag{15}$$

The bounds ϕ_{kl}, for $(k, l) \in \mathcal{I}_N \times \mathcal{I}_2$, are given in (9). Note that $\omega_{kl}(\theta) \geqslant 0, \sum_{l=1}^{2} \omega_{kl}(\theta) = 1$, for $k \in \mathcal{I}_N$.

Using expression of $\dot{\mathcal{P}}(\theta)$ in (14) and Finsler lemma [28], the following theorem provides a poly-quadratic stability analysis for LPV systems.

Theorem 2. *Consider the LPV system (10) with $\left(\theta, \dot{\theta}\right) \in \mathcal{S}_\theta \times \mathcal{S}_v$ and $\eta(\theta) \in \Xi_\theta$. If there exist symmetric matrices $X \in \mathbb{R}^{n_x \times n_x}$, $P_i \in \mathbb{R}^{n_x \times n_x}$, for $i \in \mathcal{I}_N$, and matrices $M \in \mathbb{R}^{n_x \times n_x}$, $G \in \mathbb{R}^{n_x \times n_x}$ such that the following linear matrix inequalities hold:*

$$P_i + X \succ 0, \ i \in \mathcal{I}_N, \tag{16}$$

$$\begin{bmatrix} \Psi - MA_i - A_i^\top M^\top & \star \\ P_i + X + M^\top - GA_i & G + G^\top \end{bmatrix} \prec 0, \ i \in \mathcal{I}_N, \tag{17}$$

with $\Psi = \phi_{kl}(P_k + X - P_N) + \frac{1}{N-1}\phi_{Nl} X$, for $k \in \mathcal{I}_{N-1}$ and $l \in \mathcal{I}_2$. Then, the LPV system (10) is asymptotically stable.

Note that condition (16) guarantees that function (13) is a proper candidate of Lyapunov functions for LPV stability analysis purposes. The proof of Theorem 2 is adapted from [36, Theorem 1] for the polytopic LPV setup.

Remark 4. Due to the presence of $\dot{\mathcal{P}}(\theta)$, the time-derivatives of the weighting functions $\dot{\eta}_i(\theta), \ i \in \mathcal{I}_N$, appear *explicitly* in the expression of $\dot{V}(x)$. Hence, in contrast to quadratic stability, exploiting the information on the time-varying parameter and its rate of variation plays a key role for poly-quadratic stability of LPV systems. Indeed, the bounds ϕ_{kl}, for $k \in \mathcal{I}_N$ and $l \in \mathcal{I}_2$, defined in (9), are directly involved in the stability condition (17).

Remark 5. For relaxation purposes, the decision matrices X, M and G are introduced in the stability condition (17) in Theorem 2 as slack variables. Indeed, imposing $X = 0, P_i = P \succ 0$, for $\forall i \in \mathcal{I}_N$, and $M = -P$, we can prove that the result of Theorem 2 precisely includes that of Theorem 1, see also Remark 3. With an appropriate selection of the slack variables, we can also theoretically prove that the stability condition in Theorem 2 leads to less conservative results than the classical poly-quadratic stability result in [2, Ch. 2]. Moreover, note that using of the slack decision variable X, the Lyapunov-related decision matrices P_i, for $i \in \mathcal{I}_N$, are not required anymore to be positive definite as most of existing parameter-dependent Lyapunov-based stability results [35].

2.3.3. Other choices of Lyapunov functions

Apart from poly-quadratic Lyapunov functions, other classes of Lyapunov functions have been proposed in the literature to overcome the conservatism issue of quadratic stability analysis.

2.3.3.1. Polyhedral Lyapunov. Functions Based on the ∞-norm of the state, this class of Lyapunov functions is constructed as follows [37,38]:

$$V(x) = \|P^\top x\|_\infty, \tag{18}$$

where $P \in \mathbb{R}^{n_x \times s}$ is a full row rank matrix. Although necessary and sufficient stability conditions can be derived using polyhedral Lyapunov functions [39], this class of Lyapunov functions is only suitable for LPV systems with arbitrarily-fast-varying parameters. Moreover, it leads to a non-convex control framework, which induces numerical difficulties.

2.3.3.2. Piecewise quadratic Lyapunov functions. This class of Lyapunov functions is defined as

$$V(x) = \max_{i=1,\dots,n} \{x^\top P_i x\}, \tag{19}$$

where $P_i \succ 0$, for $i \in \mathcal{I}_n$. Due to the presence of multiple matrices P_i, the piecewise Lyapunov function (19) can yield more relaxed stability results compared to the quadratic Lyapunov function (11). However, same to polyhedral Lyapunov functions, this class of Lyapunov functions leads to a non-convex framework for stability analysis and control design [40,41].

2.3.3.3. Homogeneous Lyapunov functions. Homogeneous polynomially parameter-dependent quadratic Lyapunov functions are constructed as

$$V_m(x, p) = x^\top P_m(p) x, \tag{20}$$

where $P_m(p) \in \mathbb{R}^{n_x \times n_x}$ is a homogeneous matricial form of degree m, *i.e.*, matrix whose entries are (real q-variate) homogeneous forms of degree m. As an extensions of poly-quadratic Lyapunov functions (13), the effectiveness in terms of conservatism reduction with respect to (13) has been demonstrated in [42,43]. However, this class of Lyapunov functions suffers two major drawbacks for LPV stability analysis: i) the parameters should vary arbitrarily fast, ii) the exponential increase of the computational burden as the degree of the homogeneous polynomial increases [44].

2.4. Gain-scheduled control laws for LPV systems

Extension of stability results to the control design is the main goal in LPV-based framework for nonlinear systems. As for the stability analysis, robust control theory plays a key role in the theoretical developments of LPV control. However, the parameter of LPV systems is real-time available for control design, which is not the case of uncertain/unknown parameter in robust control theory [33]. The incorporation of the time-varying parameter in the control laws leads to the concept of LPV gain-scheduled controllers [45,46]. We review below the four most common gain-scheduled control laws, in terms of both theoretical developments and real-world applications, existing in the literature for the LPV system (6).

2.4.1. Gain-scheduled static state-feedback control

The gain-scheduled state-feedback control is the most simple control law, whose structure is given by

$$u = K(\theta) x, \tag{21}$$

where the parameter-dependent gain $K(\theta) \in \mathbb{R}^{n_u \times n_x}$ is to be determined. Despite its easy design, this control structure requires full-state information for real-time implementation, which may not suitable for many practical situations [1].

2.4.2. Gain-scheduled static-output-feedback control

The gain-scheduled static-output-feedback (SOF) is another simple control structure, defined as

$$u = K(\theta) y, \tag{22}$$

where the parameter-dependent gain $K(\theta) \in \mathbb{R}^{n_u \times n_y}$ is to be determined. The great advantage of this control law consists in its simplicity for implementation. However, the related control formulation is *inherently* non-convex, which leads to a major

challenge in deriving design conditions that can be efficiently solved with available numerical solvers [47]. Based on S-variable approach [33], some numerically tractable designs of gain-scheduled SOF control have been proposed in the literature [48–50].

2.4.3. Gain-scheduled observer-based feedback control

For many engineering applications, the system state is not fully available for state feedback control and an SOF control cannot provide a viable control solution. In this situation, we can consider a gain-scheduled observer-based control scheme, whose structure relies on a full-order Luenberger observer of the form

$$\begin{aligned}\dot{\hat{x}} &= A(\theta)\hat{x} + B(\theta)u - L(\theta)(y-\hat{y}),\\ \hat{y} &= C(\theta)\hat{x},\end{aligned} \tag{23}$$

where $\hat{x} \in \mathbb{R}^{n_x}$ is the estimated state, and $L(\theta) \in \mathbb{R}^{n_x \times n_y}$ is the observer gain. Then, the gain-scheduled control law is defined as

$$u = K(\theta)\hat{x}, \tag{24}$$

where $K(\theta) \in \mathbb{R}^{n_u \times n_x}$ is the control gain. Note that a simultaneous design of both the observer gain $L(\theta)$ and the control gain $K(\theta)$ may not admit a convex formulation [2]. However, a separate observer-control design always results in LMI synthesis conditions [51–53].

2.4.4. Gain-scheduled dynamic-output-feedback control

Dynamic output feedback (DOF) control can be used to overcome the drawbacks of the two above control laws. The structure of a gain-scheduled DOF control law is given by

$$\begin{aligned}\dot{x}_c &= A_c(\theta)x_c + B_c(\theta)y,\\ u &= C_c(\theta)x_c + D_c(\theta)y,\end{aligned} \tag{25}$$

where $x_c \in \mathbb{R}^{n_c}$ is the state of the controller, and the matrices of appropriate dimensions $A_c(\theta)$, $B_c(\theta)$, $C_c(\theta)$, $D_c(\theta)$ are to be designed. This important class of LPV controllers has been largely studied in LPV control framework, especially from theoretical viewpoint, see [54–58] and related references. An interesting feature of DOF scheme is that in many control setups, convex solutions can be achieved with a full-order controller (25), *i.e.*, $n_c = n_x$. However, due to the presence of a rank constraint, the design of reduced-order DOF controllers ($n_c < n_x$) is inherently nonconvex [59,60].

It is important to note that the gain-scheduled SOF controller (22) and observer-based controller (23)-(24) are special cases of the DOF controller (25). Indeed, if $A_c(\theta) = 0, B_c(\theta) = 0, C_c(\theta) = 0$ and $D_c(\theta) = K(\theta)$ in (25), the SOF control law (22) can be directly recovered. Moreover, the DOF controller (25) becomes the observer-based control law (23)-(24) with $x_c = \hat{x}$ and

$$\begin{aligned}A_c(\theta) &= A(\theta) + B(\theta)K(\theta) + L(\theta)C(\theta), \quad C_c(\theta) = K(\theta),\\ B_c(\theta) &= -L(\theta)C(\theta), \quad D_c(\theta) = 0.\end{aligned}$$

A recent survey focusing on gain-scheduled DOF law for three LPV control setups (polytopic LPV synthesis, gridding-based LPV synthesis and multiplier-based LFT LPV synthesis) is given in [6].

2.5. Gain-scheduled control of polytopic LPV systems

General speaking, the procedure to design LPV controllers can be described with the following steps.

- **Step 1:** Select the form of the gain-scheduled controller, *i.e.*, state-feedback control, SOF control, observer-based control, DOF control.
- **Step 2:** Define the closed-loop LPV system with its performance specifications.
- **Step 3:** Apply a set of Lyapunov-based stability conditions, verifying the predefined specifications, to the closed-loop system.
- **Step 4:** Transform the stability conditions in Step 3 into numerically tractable design conditions, using for instance robust control tools [28,33].

For simplicity without loss of generality, we consider here in Step 1 a state-feedback law with $\mathcal{L}_2$-gain performance to illustrate step-by-step the control design of polytopic LPV systems. Within the polytopic control setup, the gain-scheduled controller (21) takes the form

$$u = \sum_{i=1}^{N} \eta_i(\theta) K_i x. \tag{26}$$

Note that controller (26) and the system in (8) share the same weighting functions. For Step 2, we define the closed-loop LPV system from the expressions of system (8) and controller (26) as

$$\dot{x} = \sum_{i=1}^{N}\sum_{j=1}^{N}\eta_i(\theta)\eta_j(\theta)\big((A_i + B_iK_j)x + E_iw\big). \tag{27}$$

For Step 3, we distinguish two specific cases for illustrations: quadratic control design and poly-quadratic control design.

2.5.1. Quadratic gain-scheduled control design

The following result is readily obtained for the closed-loop system (27) using the quadratic Lyapunov function (11).

Theorem 3. [2, Ch. 3] *Consider the LPV system (8) and the gain-scheduled control law (26). If there exist a positive-definite matrix $Q \in \mathbb{R}^{n_x\times n_x}$, matrices Y_i, for $i \in \mathcal{I}_N$, and a positive scalar γ such that the following LMI conditions hold:*

$$\Phi_{ii} \prec 0, \quad i \in \mathcal{I}_N, \tag{28}$$
$$\Phi_{ij} + \Phi_{ji} \prec 0, \quad i, j \in \mathcal{I}_N, \quad i < j, \tag{29}$$

where

$$\Phi_{ij} = \begin{bmatrix} \mathrm{He}(A_iQ + B_iY_j) & \star & \star \\ E_i^\top & -\gamma I & \star \\ C_iQ & F_i & -\gamma I \end{bmatrix}. \tag{30}$$

Then, the closed-loop LPV system (27) is quadratically stable and the $\mathcal{L}_2$-gain of the transfer $w \to y$ is smaller than γ, for all $\theta \in \mathcal{S}_\theta$. Moreover, the control feedback gains in (26) are defined as

$$K_i = Y_iQ^{-1}, \quad i \in \mathcal{I}_N.$$

The control result in Theorem 3 comes from the bounded-real lemma [28]. Indeed, substituting the closed-loop system (27) into this well-known lemma, it follows that

$$\begin{bmatrix} \mathrm{He}[P(A(\theta) + B(\theta)K(\theta))] & \star & \star \\ E(\theta)^\top P & -\gamma I & \star \\ C(\theta) & F(\theta) & -\gamma I \end{bmatrix} \prec 0. \tag{31}$$

Note that inequality (31) is *nonconvex* due to the nonlinear coupling between the Lyapunov matrix P and the control gains K_j, for $j \in \mathcal{I}_N$. Then, Step 4 is necessary for the control design. To this end, applying a congruence transformation [28]–(31) with $\mathrm{diag}(Q, I, I)$, followed by the change of variable $Y_j = K_jQ$, with $Q = P^{-1}$, it follows that

$$\sum_{i=1}^{N}\sum_{j=1}^{N}\eta_i(\theta)\eta_j(\theta)\Phi_{ij} \prec 0, \tag{32}$$

where Φ_{ij} is defined in (30). Then, from the convexity property, it is clear that conditions (28)-(29) guarantee (32).

2.5.2. Poly-quadratic gain-scheduled control design

To overcome a conservative design due to the use of quadratic Lyapunov functions in Theorem 3, the following theorem provides design conditions based on quadratic Lyapunov functions and *S*-variable approach.

Theorem 4. *Consider the LPV system (8) and the gain-scheduled control law (26). If there exist symmetric matrices $W \in \mathbb{R}^{n_x\times n_x}$, $Q_i \in \mathbb{R}^{n_x\times n_x}$, and matrices $M \in \mathbb{R}^{n_x\times n_x}$, $Y_i \in \mathbb{R}^{n_u\times n_x}$, for $i \in \mathcal{I}_N$, and positive scalars τ, γ such that the following LMIs are satisfied:*

$$Q_i + W \succ 0, \quad i \in \mathcal{I}_N, \tag{33}$$
$$\Gamma_{iikl} \prec 0, \quad i \in \mathcal{I}_N, \quad k \in \mathcal{I}_{N-1}, \quad l \in \mathcal{I}_2, \tag{34}$$
$$\Gamma_{ijkl} + \Gamma_{jikl} \prec 0, \quad i,j \in \mathcal{I}_N, \quad i < j, \quad k \in \mathcal{I}_{N-1}, \quad l \in \mathcal{I}_2, \tag{35}$$

where

$$\Gamma_{ijkl} = \begin{bmatrix} \Upsilon_{kl} - \mathrm{He}\mathcal{A}_{ij} & \star & \star & \star \\ Q_i + M - \tau\mathcal{A}_{ij} & \tau(M + M^\top) & \star & \star \\ E_i^\top & \tau E_i^\top & -\gamma I & \star \\ C_iM^\top & 0 & F_i & -\gamma I \end{bmatrix},$$
$$\mathcal{A}_{ij} = A_iM^\top + B_iY_j, \ \Upsilon_{kl} = \phi_{kl}(Q_k + W - Q_N) + \phi_{Nl}W.$$

Then, the closed-loop LPV system (27) is poly-quadratically stable and the $\mathcal{L}_2$-gain of the transfer $w \to y$ is smaller than γ, for all $\left(\theta, \dot{\theta}\right) \in \mathcal{S}_\theta \times \mathcal{S}_v$. Moreover, the control feedback gains in (26) are defined as

$$K_i = Y_i M^{-\top}, \quad i \in \mathcal{I}_N.$$

Note that condition (33) guarantees a proper choice of poly-quadratic Lyapunov function candidates. Conditions (34)-(35) guarantee the poly-quadratically stability of the closed-loop LPV system (27) and the $\mathcal{L}_2$-gain performance. The proof of Theorem 4 follows similar steps as for the quadratic control results in Theorem 3. Here, the key difference is that the *S*-variable approach is exploited in Theorem 4 to introduce the slack decision variable M into the design, enabling the decoupling between the Lyapunov matrices $P_i + X$ and the control gain matrices K_i, for $i \in \mathcal{I}_N$. This allows not only convexifying the design conditions but also reducing further the design conservatism.

Remark 6. As for the LPV stability analysis, in terms of conservatism relaxation, we can theoretically prove that the control result in Theorem 4 includes precisely that in Theorem 3. Indeed, this can be done by selecting $W = 0, M = Q_i = Q$, for $i \in \mathcal{I}_N$, and a *sufficiently* small scalar $\tau > 0$.

Remark 7. One of the main sources of conservatism for the control result in Theorem 4 is that the slack variable M is *parameter-independent*. This can be solved by modifying the control law (26) as

$$u = K(\theta)M(\theta)^{-1}x, \tag{36}$$

where $M(\theta) = \sum_{i=1}^{N} \eta_i(\theta) M_i$ and $M_i \in \mathbb{R}^{n_x \times n_x}$. Compared to the widely-used control structure (26), controller (36) allows avoiding some special structure of matrix decision variables, leading to less conservative design conditions [61,48]. However, note that such a non-polytopic control law requires a real-time inversion of a parameter-dependent matrix, which may induce numerical difficulties for control implementation, especially when the number of LPV submodels N becomes significantly large.

2.6. Multiple convex summation relaxation

For polytopic LPV control, in contrast to the stability analysis, the design conditions usually involve solving multiple convex summations, for instance the double convex summation (32). Due to the presence of the weighting functions, inequality (32) is an *infinite* LMI condition, which cannot be directly solved by numerical solvers. To convert (32) into a *finite* set of LMI constraints, the following usual decomposition can be performed [62]:

$$\sum_{i=1}^{N}\sum_{j=1}^{N} \eta_i(\theta)\eta_j(\theta)\Phi_{ij} = \sum_{i=1}^{N} \eta_i^2(\theta)\Phi_{ii} + \sum_{i=1}^{N}\sum_{j>i}^{N} \eta_i(\theta)\eta_j(\theta)\left(\Phi_{ij} + \Phi_{ji}\right). \tag{37}$$

Since $\eta(\theta) \in \Xi_\theta$, it is clear from (37) that conditions (28)-(29) are *sufficient* to guarantee (32). The *sufficiency* of multiple convex summation based conditions may cause additional conservatism to the control design. Various approaches have been proposed to reduce this source of conservatism, for instance without introducing slack variables [62,63], and with the use of slack variables [64,65]. Among these approaches, Pólya's theorem based relaxation [65] provides *asymptotically necessary and sufficient* LMI-based conditions to check the definite positiveness of multiple convex summations. However, such a relaxation result is only meaningful from the theoretical viewpoint since the computational burden *exponentially* increases with respect to the homogeneous degree of the summations [66].

2.7. Polytopic LPV models and Takagi–Sugeno models

Takagi–Sugeno (T-S) fuzzy model-based approaches have been known as an effective framework to deal with nonlinear systems [27,66]. T-S fuzzy modeling, first proposed in [67], is expressed by fuzzy IF-THEN rules which represent local dynamics of nonlinear systems as

$$\begin{aligned} \text{Rule } R_i: \quad & \text{If } z_1 \text{is } \mathcal{M}_1^i \text{ and } \cdots \text{ and } z_p \text{is} \mathcal{M}_p^i, \\ & \text{Then } \dot{x}(t) = A_i x + B_i u + E_i w, \end{aligned} \tag{38}$$

where R_i denotes the ith fuzzy inference rule, N is the number of inference rules, $\mathcal{M}_j^i$, with $i \in \mathcal{I}_N$ and $j \in \mathcal{I}_p$, are the fuzzy sets, and (A_i, B_i, E_i) the state-space matrices of appropriate dimensions of the ith local linear sub-model. The vector of premise variables is defined as $z = \left[z_1 \cdots z_p\right]^\top$. Using the center-of-gravity method for defuzzification, the T-S fuzzy model (38) can be rewritten in the compact form

$$\dot{x} = \sum_{i=1}^{N} h_i(z)(A_i x + B_i u + E_i w),$$

where the membership function $h_i(z)$ is defined as

$$h_i(z) = \frac{\omega_i(z)}{\sum_{i=1}^{N}\omega_i(z)}, \quad \omega_i(z) = \prod_{j=1}^{p}\mu_j^i(z_j), \quad i \in \mathcal{I}_N.$$

The grades of membership of the premise variables in the corresponding fuzzy sets $\mathcal{M}_j^i$ are given as $\mu_j^i(z_j)$. Note that the normalized membership functions satisfy the convex sum property as [27]:

$$0 \leqslant h_i(z) \leqslant 1, \quad \sum_{i=1}^{N} h_i(z) = 1, \quad \sum_{i=1}^{N} \dot{h}_i(z) = 0. \tag{40}$$

From their respective expressions that polytopic LPV system (8) and T-S fuzzy system (39) share some analogies [68]. Specifically, the scheduling parameter θ of LPV system (8) corresponds to the premise variable z of T-S system (39). Moreover, these systems are constructed by "merging" linear submodels together with the weighting functions $\eta_i(\theta)$ for LPV systems or the membership functions $h_i(z)$ for T-S fuzzy systems. Hence, T-S fuzzy systems can be considered as polytopic quasi-LPV systems. Perhaps the major difference between polytopic LPV and T-S fuzzy systems consists in their historical backgrounds, *i.e.*, robust control theory in LPV case and fuzzy theory in T-S case.

Due to the strong analogies, a large number of theoretical tools can be applied to both polytopic LPV systems and T-S fuzzy systems, especially Lyapunov method in conjunction with LMI techniques [28]. However, note that within T-S fuzzy framework the premise variables generally depend on the state vector. Then, the time-derivatives of the membership functions, depending on the time-derivative of the state, are generally not available for control design of system (39). This implies much more numerical and theoretical challenges when using poly-quadratic Lyapunov functions, also called fuzzy Lyapunov functions [69], for stability analysis and control design of continuous-time T-S fuzzy systems. Indeed, most of results are formulated using local analysis settings with different degrees of conservatism, see [66] for a recent discussion.

3. Applications to vehicle dynamics control

This section provides some polytopic LPV-based results on vehicle dynamics control. Different suspension models, descriptions of vehicle lateral dynamics and integrated vehicle models used for control design are briefly discussed. Then, polytopic LPV controller design methods with various settings of scheduled parameters are summarized.

3.1. Vehicle dynamics

3.1.1. Vehicle suspensions

The main functions of a well-designed suspension system can be summarized as follows.

- *Ride quality*. A vehicle suspension has a function of providing an isolation by decreasing forces transmitted from the vehicle axle to the vehicle body. The accelerations of the sprung mass are used as a performance indicator.
- *Suspension deflection limit*. An excessive suspension bottoming should be avoided. Hence, the suspension stroke should be constrained to a prescribed level.
- *Road holding*. The wheels and the road should contact uninterruptedly. Moreover, the dynamical tire load should not exceed the static one.

For the design of suspension systems, three categories of suspension models, including full-car models [70–74], half-car models [75,76], and quarter-car models [77–85], have been widely used. According to different control strategies, the designed suspensions can also be divided as active suspensions [71,72,75–77,79,81,82,86–88] and semi-active suspensions [73,74,78,80,83,84,89,90].

3.1.1.1. Active suspension. Let us consider the half-car model depicted in Fig. 2. Assume that the structure of the vehicle is symmetrical, the pitch angle is small, and all the springs and dampers have linear dynamics. As a result, the half-car model can be described as follows [75]:

$$\begin{aligned}
M\ddot{z}_c &= f_f + f_r,\\
J\ddot{\theta} &= af_f - bf_r,\\
m_f\ddot{z}_f &= -k_{f2}\left(\eta_f - \mu_f\right) - f_f,\\
m_r\ddot{z}_r &= -k_{r2}\left(\eta_r - \mu_r\right) - f_r,
\end{aligned}$$

where M and J denote the sprung mass and the mass moment of inertia, m_f and m_r stand for the front and rear unsprung masses, respectively, u_f and u_r indicate the control forces of the active strategy, k_{f1} and k_{r1} represent the stiffness coefficients of the passive suspension elements, b_f and b_r are the damping coefficients of the passive suspension elements for the front and rear assembles, k_{f2} and k_{r2} represent the front and rear tire stiffness, and

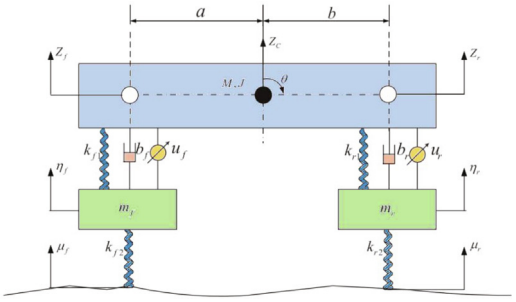


Fig. 2. Schematic diagram of a half-car active suspension system.

$$\begin{aligned} f_f &= k_{f1}\left(\eta_f - z_c - a\theta\right) + b_f\left(\dot{\eta}_f - \dot{z}_c - a\dot{\theta}\right) + \mu_f, \\ f_r &= k_{r1}(\eta_r - z_c + b\theta) + b_r\left(\dot{\eta}_r - \dot{z}_c + b\dot{\theta}\right) + \mu_r. \end{aligned} \tag{41}$$

The displacements at the front and rear wheels can be calculated as follows:

$$z_f = z_c + a\theta, \quad z_r = z_c - b\theta. \tag{42}$$

Similarly, full-car models and quarter-car models can be developed [71,77], which are omitted here for brevity.

3.1.1.2. Semi-active suspension. For semi-active suspension systems, as shown in Fig. 3 for a quarter-car suspension model, there is no component to produce the control force. Variable damper or other variable dissipation components are used to modify the damping coefficient. Similar to active suspensions, a semi-active suspension system of a quarter-car can be modeled as

$$\begin{aligned} &m_s\ddot{z}_s + b_s(\dot{z}_s - \dot{z}_u) + k_s(z_s - z_u) = -b_{semi}(z_s - z_u), \\ &m_u\ddot{z}_u + c_s(\dot{z}_u - \dot{z}_s) + k_s(z_u - z_s) + k_t(z_u - z_r) = b_{semi}(\dot{z}_s - \dot{z}_u), \end{aligned} \tag{43}$$

where b_{semi} is the variable damper to be regulated.

3.1.2. Vehicle lateral dynamics

For the control of vehicle lateral dynamics, the classical bicycle model is mostly used, as shown in Fig. 4. A simplified bicycle model consists of two degrees of freedom, *i.e.*, lateral and yaw motions, of which the dynamics can be described as

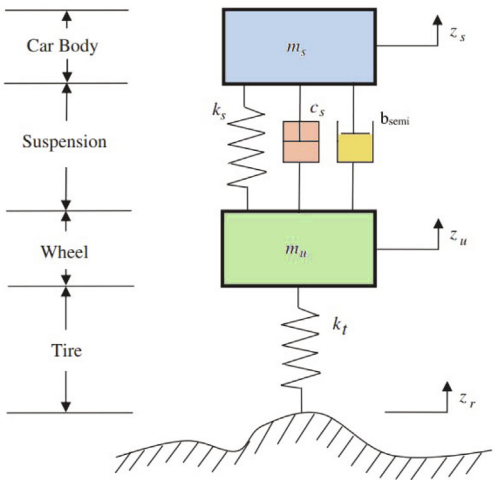


Fig. 3. Schematic diagram of a quarter-car semi-active suspension system.

$$\begin{aligned} mV_x\left(\dot{\beta}+\gamma\right) &= F_{yf}+F_{yr},\\ I_z\dot{\gamma} &= l_fF_{yf}-l_rF_{yr}+M_z, \end{aligned} \tag{44}$$

where m denotes the mass of the vehicle, V_x is the longitudinal vehicle velocity, β stands for the sideslip angle, γ represents the vehicle yaw rate, F_{yf} and F_{yr} denote the lateral tire forces of front and rear wheels, respectively. I_z stands for the moment of inertia around the vertical axis, l_f and l_r denote the distance between the center of gravity and the front and rear axis. In addition, M_z is the external yaw moment.

For normal driving conditions [91], the lateral tire forces in the vehicle model (44) can be modeled as

$$F_{yf}=C_{yf}\alpha_f,\quad F_{yr}=C_{yr}\alpha_r,$$

where C_{yf} and C_{yr} are the cornering stiffness of front and rear tire, respectively. The tire slip angles of the front and rear tires can be respectively represented as

$$\alpha_f=\delta-\frac{l_f\gamma}{V_x}-\beta,\quad \alpha_r=\frac{l_r\gamma}{V_x}-\beta.$$

Then, the vehicle lateral dynamics can be given by the following state-space model as

$$\dot{x}=Ax+B_1u+B_2w, \tag{45}$$

where $x=[\beta\ \gamma]^\top, u=M_z, w=\delta$, and

$$A=\begin{bmatrix} -\frac{C_{yf}+C_{yr}}{mV_x} & \frac{C_{yr}l_r-C_{yf}l_f}{mV_x^2}-1\\ \frac{C_{yr}l_r-C_{yf}l_f}{I_z} & -\frac{C_{yf}l_f^2+C_{yr}l_r^2}{I_zV_x}\end{bmatrix},\ B_1=\begin{bmatrix}0\\ \frac{1}{I_z}\end{bmatrix},\quad B_2=\begin{bmatrix}\frac{C_{yf}}{mV_x}\\ \frac{C_{yf}l_f}{I_z}\end{bmatrix}.$$

Note that, when the longitudinal-lateral dynamics coupling is taken into account for controller/observer design, it is possible that the lateral velocity V_y and the yaw rate γ are considered together with the longitudinal velocity V_x as the state variables to form the corresponding nonlinear vehicle models [92–94].

3.1.3. Integrated vertical and lateral dynamics

The control issues of integrated vehicle dynamics have become a research hotspot in recent years. The control performance of the vehicle dynamics can be greatly improved by integrating the active chassis control of active steering, active suspension, and active braking. The authors in [95] proposed a multi-variable design strategy for chassis control including active steering, electro-mechanical braking actuators, and semi-active suspension. The designed LPV controller leads to an significant performance improvement in critical driving situations. Similarly, suspension systems were considered together with braking systems [86,87,90], steering systems [76,86,90,96], and electronic stability program (ESP) systems [72,87] to enhance the vehicle control performance.

Considering a quarter-car active suspension model together with a vehicle lateral dynamics model, we can obtain the following integrated vehicle model including vertical and lateral dynamics [76]:

$$\dot{x}=A(\rho)x+B_1u+B_2w, \tag{46}$$

where $x=\left[\beta\ \gamma\ \dot{z}_c\ \dot{\theta}\ \dot{z}_f\ \dot{z}_r\ \dot{\eta}_f\ \dot{\eta}_r\ z_f\ z_r\ \eta_f\ \eta_r\right]^\top, u=[\delta_u\ u_f\ u_r]^\top$ and $w=\left[\delta_c\ \mu_f\ \mu_r\right]^\top$. δ_c and δ_u denote the nominal steering angle and the assistant steering angle for the front wheel. The matrices $A(\rho)$, B_1 and B_2 in (46) can be found in [76].

Another integrated vehicle control method consists in decomposing the global chassis control into two steps [95]. In the first step, the linear bicycle model is analyzed and a controller is designed to improve the vehicle lateral stability. In the second step, a suspension controller is synthesized to enhance the vehicle vertical performance.

3.1.4. Integrated lateral, longitudinal and braking dynamics

Taking into account the braking system, the vehicle dynamics system can be given as follows [97]:

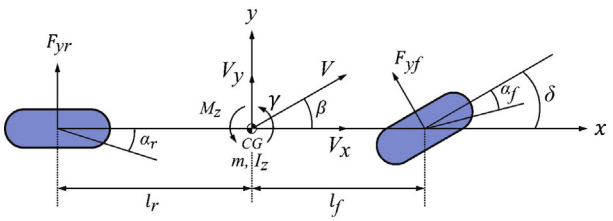


Fig. 4. Bicycle model for vehicle lateral dynamics.

$$\begin{bmatrix} \dot{x} \\ z \\ y \end{bmatrix} = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ w \\ u \end{bmatrix}, \tag{47}$$

where $x = [\beta \quad \gamma]^\top$, $w = [\gamma_d \quad \beta_d \quad M_{zd}]^\top$ is the disturbance input, $u = [\delta \quad T_{brl} \quad T_{brr}]^\top$ denotes the control input, $y = [\gamma \quad \beta]^\top$ stands for the measured output, and $z = [z_1 \quad z_2 \quad z_3 \quad z_4]^\top$ represents the controlled output. Note that z_1 denotes the weighted yaw rate error output signal, z_2 stands for the weighted sideslip angle error output signal, z_3 represents the braking control signal attenuation, z_4 stands for the steering control signal attenuation. The state-space matrices A, B_1, B_2, C_1, D_{11}, D_{12} and C_2 of the vehicle model (47) can be found in [97].

3.1.5. Observer-based vehicle models

Since full-state information of vehicle systems are generally not available online due to the issue of sensor costs. For instance, the vehicle sideslip angle cannot be reliably measured with low-cost sensors in practice. Hence, LPV observers have been proposed to estimate important vehicle variables [35,98,99]. To design an LPV observer for estimating the vehicle sideslip angle, $\frac{1}{V_x}$ and $\frac{1}{V_x^2}$ were chosen as time-varying parameters of the LPV vehicle system in [98]. For observer design, a triangular polytope was derived to reduce the number of vertices of the polytopic LPV vehicle model. Using Lyapunov stability method, LMI-based design conditions were developed with an augmented vehicle model, including the estimation error dynamics and the vehicle lateral dynamics. Moreover, an energy-to-peak gain specification was taken into account in the observer design to improve the estimation performance under unknown disturbances. Similarly, a robust sideslip angle observer was proposed in [99] for electric ground vehicles based on a lateral vehicle model and the measurement of the yaw rate. The uncertain tire characteristics were taken into account in the observer design. Since the tire cornering stiffness and the vehicle inertial moment cannot be easily measured in practice, a parameter identification procedure was provided with experimental data. The vehicle state and the driver torque are simultaneously estimated in [35] using an LPV observer with an unknown input. To derive the polytopic LPV vehicle model, $\frac{1}{V_x}$ was chosen as the scheduling variable and $\frac{1}{V_x^2}$ is approximated via the first-order Taylor's approximation as

$$\frac{1}{V_x} = \frac{1}{V_0} + \frac{1}{V_1}\theta, \quad \frac{1}{V_x^2} = \frac{1}{V_0^2}\left(1 + 2\frac{V_0}{V_1}\theta\right), \tag{48}$$

where θ is used to describe the variation of V_x between its lower bound $V_{\min}$ and upper bound $V_{\max}$ with $-1 \le \theta \le 1$. The terms V_0 and V_1 in (48) are given by

$$V_0 = \frac{2V_{\min}V_{\max}}{V_{\min} + V_{\max}}, \quad V_1 = \frac{-2V_{\min}V_{\max}}{V_{\max} - V_{\min}}. \tag{49}$$

Based on lateral-longitudinal integrated dynamics, a nonlinear LPV unknown input observer was developed in [94] for a *simultaneous* estimation of the lateral speed, the steering input and the effective engine torque. The speed-related term $\frac{1}{V_x}$ and the yaw rate γ were used for gain-scheduling purposes.

3.2. Choices of scheduling parameters for LPV control

The aforementioned vehicle models can be represented in affine/polytopic LPV forms. Different affine/polytopic LPV models have been established to handle various vehicle control and/or estimation problems. Depending on the considered time-varying parameters, the vehicle models can be reformulated in a general form as follows:

$$\dot{x} = A(\rho)x + B_1(\rho)u + B_2(\rho)w, \tag{50}$$

where $A(\rho)$, $B_1(\rho)$ and $B_2(\rho)$ are state-space matrices obtained according to the time-varying variables of vehicle systems. Table 1 summarizes the different choices of time-varying variables in vehicles dynamics control and estimation, discussed hereafter.

3.2.1. Time-varying velocity

Since the vehicle velocity changes during cruising in practice, its related parameters could be regarded as scheduling parameters [75,76,108,116,117,126]. In addition, the vehicle lateral acceleration can be also chosen as a scheduling parameter [93]. Since the vehicle velocity is inherently time-varying, the parameters $\rho_1 = V_x$, $\rho_2 = \frac{1}{V_x}$, $\rho_3 = \frac{1}{V_x^2}$ are considered as scheduling parameters in [76]. Then, an integrated vehicle dynamics control strategy is proposed to enhance the vehicle safety and handling performance by combining active front steering and active suspension systems. A polytopic LPV vehicle model with finite vertices, affinely depending on the time-varying longitudinal speed, is built. Then, an $\mathcal{H}_\infty$ gain-scheduled controller is designed using LMI-based techniques and Lyapunov stability arguments. Considering $\rho_1 = V_x$ and $\rho_2 = \sqrt{V_x}$ as scheduling parameters, a velocity-dependent multi-objective LPV control method is presented in [75] to solve the preview control problem with velocity uncertainty. Similarly, $\rho = V_x$ is considered as a scheduling parameter for the LPV control

Table 1
LPV Approaches for vehicle dynamics control and estimation.

Time-varying parameters	Suspension model	Lateral model	Integrated model	Observer design
Vehicle velocity	[75]	[100–115]	[76,116–118]	[98,99,35]
Deflection-related parameters	[70,71,77,84]	–	[95]	–
Braking-related parameters	–	–	[86,87,90,95,119]	–
Cornering stiffness	–	[120]	–	–
Other choices of parameters	[73,74,78–81,83]	[108,112,114,115,121,122]	[72,96,123–125]	[74,88,89]

design in [116]. In this work, a variable geometry suspension and a robust suspension control are both considered to enhance the vehicle stability.

The velocity related terms $\frac{1}{V_x}$ and $\frac{1}{V_x^2}$ are considered as scheduling parameters in [100–107,127]. With different definitions of vehicle state vectors, $\rho_1 = V_x$, $\rho_2 = \frac{1}{V_x}$ and $\rho_1 = V_x$, $\rho_2 = \frac{1}{V_x}$, $\rho_3 = \frac{1}{V_x^2}$ are adopted as the scheduling parameters in [109,118], respectively. In these polytopic LPV settings, the constructed polytope would have 2^n vertexes, where n is the number of scheduling parameters. For example, if the $\rho_1 = \frac{1}{V_x}$, $\rho_2 = \frac{1}{V_x^2}$, then a quadratic polytope will be obtained as depicted in Fig. 5(a). However, since the varying parameters are velocity-related, which are not independently variant, some methods are proposed to shrink the polytope in order to reduce the condition conservatism. The trapezoidal polytope in [103,106,118], triangle polytope in [100], and two-vertices form in [101,104,127] are proposed as shown in Figs. 5(b)–(d), where the two vertices are given as

$$\bar{\Omega}_1 = \left(\frac{1}{\overline{V}_x}, \frac{7\underline{V}_x^2 + 2\underline{V}_x\overline{V}_x - \overline{V}_x^2}{8\underline{V}_x^2\overline{V}_x^2} + \frac{\left(\overline{V}_x - \underline{V}_x\right)^2}{8\underline{V}_x^2\overline{V}_x^2}N_2(t)\right), \bar{\Omega}_2 = \left(\frac{1}{\underline{V}_x}, \frac{7\overline{V}_x^2 + 2\underline{V}_x\overline{V}_x - \overline{V}_x^2}{8\underline{V}_x^2\overline{V}_x^2} + \frac{\left(\overline{V}_x - \underline{V}_x\right)^2}{8\underline{V}_x^2\overline{V}_x^2}N_2(t)\right),$$

with $|N_2(t)| \leqslant 1$. Based on the constructed LPV models, $\mathcal{H}_\infty$ control [109], $\mathcal{H}_\infty$ control in the μ-split problem [118], multiobjective energy-to-peak control design with $\mathcal{D}$-stability [101], double layer control strategy considering a tire-force saturation [100,102], $\mathcal{H}_\infty$ state-feedback with state delay [105], $\mathcal{H}_\infty$ output-feedback control [106], have been investigated.

Apart from the above polytopic reductions, another two-vertices formulation could be found in [49,109,126]. To this end, a change of time-varying parameter is performed for $\frac{1}{V_x}$ together with a first-order Taylor approximation as in (48).

3.2.2. Suspension-deflection-related parameters

Due to the motion of suspension systems, the suspension deflection and its velocity could be regarded as time-varying parameters [70,77,95]. Moreover, the authors in [71,77] also consider the influences of some other varying parameters besides the suspension deflection. For instance, the variations in the suspension deflection and mass are defined as scheduling parameters to form an LPV model in [71]. The resulting LPV controller enables the vehicle suspension systems to prevent from hitting their structural limits. In [77], the suspension deflection and the time-varying parameter representing the road conditions are used for gain-scheduled control purposes. The derived LPV controller allows minimizing either the acceleration or the suspension deflection, directly depending on the magnitude of the suspension deflection.

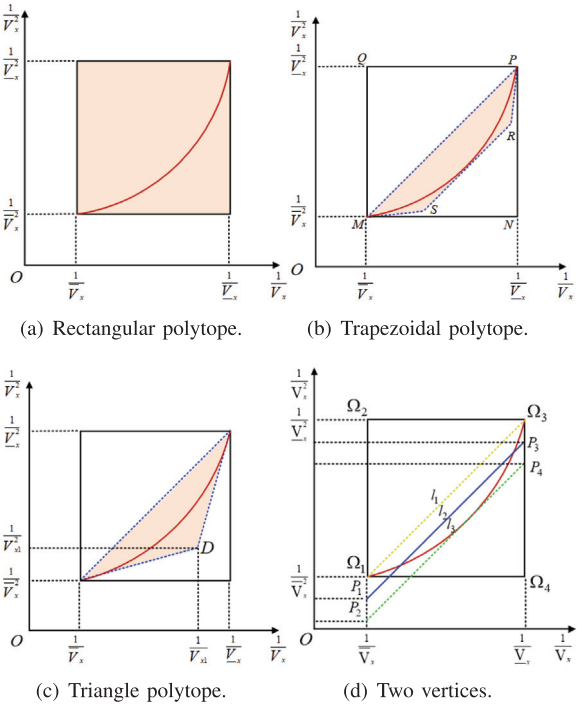
3.2.3. Braking-monitor-related parameters

The braking monitor parameters are considered as time-varying parameters in [86,87,95]. In [95], an LPV control strategy is proposed to enhance the vehicle performance in critical driving situations. The scheduling parameters R_b (braking monitor) and R_s (suspension and steering monitor) are used to analyze the system and design three controllers for steering, braking and semi-active suspension. The parameters R_b and R_s are also chosen as the scheduling parameters in [87]. In the work, the proposed control strategy leads to the situation dependent objectives in a unified framework. Due to the adaption of R_b and R_s parameters, the control performance is smooth, while satisfying internal stability and minimizing an $\mathcal{L}_2$-gain performance.

Furthermore, a new $\mathcal{H}_\infty$ LPV coordination strategy which aims to improve the vehicle stability using active steering, suspension, and electro-mechanical braking actuators was proposed in [86]. The main idea of this coordination technique is to tune the suspensions in the four corners and to improve the vertical performance by measuring the load transfer distribution of the vehicle while the vehicle is running on irregular road. The results showed that the proposed strategy has a good coordination between braking and steering actuators.

3.2.4. Braking-efficiency-related parameters

The braking efficiency and its related weighting function were considered as scheduling parameters in [90,119,97,123–125]. Some of these works have considered the actuator coordination with different parameters. As for [97,123,125], to coordinate the steering and braking actuators in different situations, a weighting function based approach has been used. However, there are some differences among them. The authors in [97,123] constructed two parameters ρ_1 and ρ_2 to continuously (de) activate the steering and the braking actions. Moreover, the activated rear braking actuator was selected according to the values of ρ_1 and ρ_2 (either 0 or 1). The key difference is that ρ_1 in [97] can only be 0 or 1 while this parameter can be



(a) Rectangular polytope. (b) Trapezoidal polytope.

(c) Triangle polytope. (d) Two vertices.

Fig. 5. Different types of polytopes representing $\left[\frac{1}{V_x}, \frac{1}{V_x^2}\right]$.

continuously changed between 0 and 1 in [123], allowing for a smoother activation of the steering actuator. The authors in [125] constructed a weighting function of the braking control signal according to a scheduling variable $\rho \in \left[\underline{\rho}, \overline{\rho}\right]$. When the time-varying parameter ρ increases, the braking input is penalized. On the contrary, if ρ decreases, no penalization is performed for the braking control signal.

Different from the three previous papers, the dynamics and the effects of the suspension were also considered in [90,119,124]. Similar to the work in [125], the authors in [119] also used a weighting function to achieve the steering-braking coordination. However, the difference is that a function of the braking efficiency according to the scheduling variable $\xi \in \left[\underline{\xi}, \overline{\xi}\right]$ was used in [119]. Then, depending on a larger or a smaller value of ξ, the steering input is penalized or not. The authors in [90] proposed two scheduling parameters: $R_b \in [0, 1]$ (braking) and $R_s \in [0, 1]$ (suspension and steering). When the values of these two parameters *gradually* decreases from 1 to 0, the system transits from a normal situation to an intermediate or even a critical situation. The problem of fault tolerant control of a semi-active suspension system was considered in [124]. Using a time-varying parameter $\alpha \in [0, 1]$, estimated by a fault detection and diagnosis strategy, the authors proposed an online adjustment of the semi-active damper in case of leakage. Based on the constructed LPV models with different choices of time-varying scheduling parameters, robust $\mathcal{H}_\infty$ gain-scheduled control methods were studied in [97,123,125], and $\mathcal{H}_\infty$ LPV global chassis control methods considering the suspension dynamics were investigated in [90,119,124].

3.2.5. Other choices of scheduling parameters

Apart from the varying longitudinal velocity, the varying lateral velocity, varying yaw rate and the difference between desired and actual transmitted yaw torque are also considered in [110–113], respectively. The fault-tolerant control is proposed with $\rho_1 = V_x,\ \rho_2 = V_y,\ \rho_3 = \frac{1}{V_x},\ \rho_4 = \frac{V_y}{V_x^2}$; and $\rho_1 = V_x,\ \rho_2 = \frac{1}{V_x},\ \rho_3 = r$, respectively. In [120], the cornering stiffness C_{yf} and C_{yr} are chosen as the varying parameters and the four-vertices rectangular polytope is used to describe them, and a state-feedback multi-objective control is proposed to enhance the vehicle lateral stability. In [121,114], the tire slip angles α_f and α_r are taken into account as varying parameters, but the difference is [114] also considering the longitudinal velocity V_x as a varying parameter, [121] considering the tire slip angles $\alpha_f(\lambda), \alpha_r(\lambda)$ as a function of adhesion coefficient λ. Moreover, the authors in [115] consider the steering angle δ as a scheduling parameter. However, to avoid the singular point when $\delta \to 0$, a change of variable has been done by shifting the δ interval: $\delta \in \left[\underline{\delta}, \overline{\delta}\right] \to \sigma \in \left[\underline{\delta} + \varepsilon, \overline{\delta} - \varepsilon\right]$, converting σ into the new scheduling variable and being ε a constant value greater than δ.

4. Applications to autonomous vehicles

Besides the traditional vehicle dynamics control, the LPV techniques have successfully applied to the path following control problem [128–130] and lateral tracking control [131] for autonomous vehicles. The path following control problem is one of the fundamental challenges for the development of autonomous vehicles. Different from the vehicle dynamics control, there are vehicle–road model, preview model, vehicle dynamics model, and steering system model for the path following problem. Therefore, the design is more challenging.

The authors in [132] studied the path following problem of autonomous ground vehicles via output-feedback control and robust $\mathcal{H}_\infty$ technique. Both the variations of the longitudinal velocity and the cornering stiffness were considered. Based on similar model in [132], the authors in [133] investigated the network-induced delay and data dropouts in path following control problem. The tracking performance can be guaranteed when there is a bounded delay in the steering control input. The research team in [134,135] considered different driver steering characteristics for an advanced driver assistant system belonging to a low level autopilot. The authors in [136] developed a specific LPV lateral motion model of unmanned ground vehicles and proposed a robust gain-scheduled automatic steering controller design method to exploit with the time-varying velocity and external disturbance. With friction force estimation and compensation mechanism, the authors in [137] designed a gain-scheduling controller for autonomous vehicles. The model predictive approach was employed to act as a path planner in [138]. Based on the planned trajectory, a tracking controller was designed with the LPV technique. The input saturation constraint was considered for steering control design in [139]. An output feedback gain-scheduled control design was proposed for the path following of autonomous ground vehicles [140], for which the closed-loop transient performance can be improved via the concept of $\mathcal{D}$-stability. In the work [141], the autonomous vehicle has a four-wheel steering system and a four-wheel driving system. A robust $\mathcal{H}_\infty$ controller was designed via the LPV technique for the specific vehicle. The gain-scheduled composite nonlinear feedback control method was utilized to facilitate an impaired driver assistance system to achieve better trajectory tracking performance in [142].

5. Applications to vehicular powertrain systems

The powertrain system in which the main function is to convert the power to movement of rotating wheels is one of the most important parts for vehicles. In terms of the power source, there are engine-powered vehicles, motor-powered vehicles, and hybrid vehicles. The powertrain of an engine-powered vehicle generally consists of the engine, clutch, transmission, drive shaft, final drive, and wheels. The powertrain of a motor-powered vehicle is composed of a motor, a motor drive system, a DC–DC converter, a transmission system, and a battery system. In a hybrid vehicle, the vehicle is driven by an electrical motor and an internal combustion (IC) engine. Though the powertrain systems are different, the nonlinearity in the model is a common challenge for powertrain control design. The review of LPV-technique applications on vehicular powertrain systems focuses on the control and estimation of typical powertrain components such as engines, aftertreatment systems, and electric vehicles.

5.1. Internal combustion engines

The control design for IC engines is known as a challenging problem due to the modeling complexities and the involved nonlinearities [143]. Up to now, many control strategies have been proposed for IC engines. Especially, LPV techniques have been broadly applied in the last decades [25,144–146]. The simplified diagram of a powertrain system in IC engines is described in Fig. 6.

For gasoline engines, LPV technique has been mainly applied to the air–fuel ratio control problem. Feng et al. designed an air–fuel feedback controller by applying LPV control for a spark ignition (SI) engine [144]. This paper aims to solve the problems of variable time delay in the system and thus maximize the total fuel economy. Marius et al. constructed an LPV model and developed a gain-scheduled strategy to control the air–fuel ratio in SI engines [147]. In this controller, the engine speed and air flow were regarded as the scheduling parameters to achieve the optimal control. A switching LPV controller was proposed in [148] for air–fuel ratio control of SI engines. The authors represented the system dynamics as a first-order LPV model, which is effective to describe the change of the engine operating points. Then, the control problem was solved through LMI constraints, and the performance of the proposed LPV controller was demonstrated under various simulation tests. Furthermore, LPV control technique was also used for air charge control of gasoline engines in [149]. Abbas et al. first built a quasi-LPV model based on neural state-space model, then developed a discrete-time LPV controller to carry out the air charge control for an SI engine. Similarly, the optimization problem was solved based on LMI constraints and an evolutionary search. A fixed-structure LPV control was also proposed for air charge control of SI engines in [145]. For the air charge control, the authors combined an affine LPV model together with a gain-scheduled PID controller, which allows achieving a good control performance in real-time experiments. An LPV input–output model was proposed in [150] to represent the nonlinear dynamics of air path systems in turbocharged SI engines. Then, LPV control technique was applied to perform the engine charge control. Except for the air–fuel ratio control and the air charge control, the wastegate control was investigated for turbocharged SI engines in [151]. Quasi-LPV modeling was explored to describe the engine nonlinear dynamics, then an internal model control design was proposed to achieve the boost-pressure tracking. LPV control technique was also

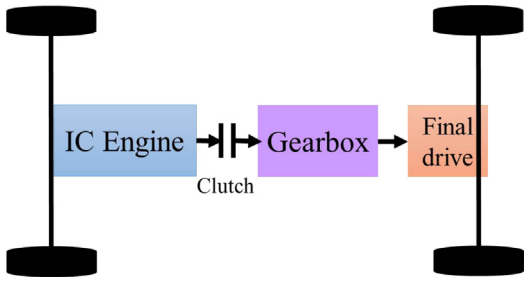


Fig. 6. Schematic diagram of a powertrain system in IC engines.

employed to implement the total engine optimization for SI engines [152]. In this work, a hierarchical control structure, including both upper-level and lower-level controllers, was proposed. Model predictive control framework was adopted as the upper-level controller, and the lower-level controller was designed using LPV control. Both simulations and real track tests were presented to demonstrate the superiority of the developed method.

LPV control methods have been also exploited to address different control problems in Diesel engines. LPV techniques were used to deal with the modeling and control issues of air path systems in Diesel engines [153,154]. The authors developed a data-driven grey-box model based on quasi-LPV framework to represent the system behaviors, and further designed a gain-scheduled $\mathcal{H}_\infty$ controller to improve the tracking performance. The experimental results showed that the proposed controller is able to control effectively the transient exhaust gas fraction. Based on a physical model, Lihua et al. [155] investigated a quasi-LPV control method to manage the air path system of Diesel engines. To tackle the challenge due to the strong nonlinearities of Diesel air path systems, a Hammerstein quasi-LPV model was used to approximate the system dynamics, then a gain-scheduled law was considered for control design. Besides, an LPV air path model of a turbocharged Diesel engines was studied in [156]. Based on this three-order LPV model, a robust gain-scheduled controller was proposed, which allows reducing significantly the calibration effort. The regulation of the Diesel engine speed was studied through LPV control approach in [157]. This work aims at computing optimal control inputs under fast operating condition changes of Diesel engines and variable transport delays. The performance of the designed controller was validated with hardware-in-the-loop tests. Boost-pressure control is also an important issue in Diesel engines. The authors in [158] explored LPV technique to address the control problem of air path systems. The performance of the proposed LPV method was experimentally validated with an BMW Diesel engine. In [159], the control problem of a common rail injection system was investigated via an $\mathcal{H}_\infty$ LPV control framework. The main contribution of this work was the system modeling through LFT representation under time-varying engine speed, rail pressure and fuel temperature. Fresh air fraction control was promoted based on LPV hyperbolic systems for Diesel engines in [160]. By stabilizing the LPV hyperbolic system with boundary conditions, the optimal air mass fraction can be obtained, which leads to a good control performance for Diesel engines.

Although most of the works concerning LPV techniques in conventional engines are related to controller design, some investigations on LPV estimation were also carried out in past decades. Zhiyuan et al. proposed an LPV adaptive observer to jointly estimate the system states and parameters in a Diesel engine [161]. This observer is able to compensate the mass air flow sensor error and update an error map online. Finally, the simulation results proved the developed observer is quite qualified for the sensor error compensation. LPV modeling was also applied for fault detection and isolation (FDI) in turbocharged SI engines [162]. The LPV model in this work was mainly used to approximate the nonlinear dynamics of SI engines. Then, $\mathcal{H}_\infty$ Luenberger observers were developed to construct the FDI architecture. Lastly, numerical simulations showed the effectiveness of the proposed LPV model and the designed observers.

5.2. Electric vehicles

Considering the environmental problems, electric vehicles including pure electric vehicles and hybrid electric vehicles (HEVs) would dominate the future worldwide market of ground vehicles, and traditional vehicles equipped with IC engines would be gradually eliminated. For electric vehicles, the powertrain control has been always a hot research topic. In particular, LPV control methods for electric powertrain systems have attracted increasing attention.

The scholars from Hamburg University of Technology focused on the investigation of torque vectoring in electric vehicles using LPV techniques, and published several valuable papers [163,164]. In their work, LPV gain-scheduled controllers were developed based on quadratic Lyapunov functions to track the longitudinal velocity and yaw rate of the vehicle. Besides, torque and slip limiters were considered to address the physical saturation problem in electric motors and wheel slip constraints in the control design. Energy management using LPV control technique is also an appealing idea in electric vehicles. Waleed et al. mainly studied the energy management of electric vehicles with variable power source-fuel cell, battery and ultra-capacitor [165–167]. An $\mathcal{H}_\infty$ gain-scheduled controller associated to weighting functions was proposed to regulate the fuel cell current and the converter voltage, and to further achieve the optimal power output of the electrical motors. Additionally, the weighting functions in LPV control were selected through a genetic algorithm. Different driving cycles con-

ducted in MATLAB/SIMULINK showed the good performance of designed controller. LPV strategy was adopted to control the permanent magnet synchronous motor (PMSM) for electric vehicles in [168]. A parameter-dependent Lyapunov function was employed for LPV control design to guarantee the robust stability under time-varying parameters. The simulation results illustrated that the controller could work well in *d-q* rotating frames.

Four wheel independently-actuated (FWIA) electric vehicle has attracted increasing attention from researchers due to the actuation flexibility. As an FWIA electric vehicle is driven through in-wheel motors, the torque control of FWIA electric vehicle is regarded as a type of powertrain control. LPV control is widely used for FWIA electric vehicle, for example, Wang et al. developed an LPV control strategy to ensure the stability and improve the handling of FWIA vehicles [111]. In this article, a novel linear quadratic regulator (LQR)-based controller considering different actuator faults was designed by using LPV techniques. Through the LQR-based LPV control, the negative effect result from actuator faults and external disturbances is minimized. Similarly, robust fault-tolerant control based on LPV techniques was proposed to perform the trajectory tracking control of FWIA vehicles [169]. When faults occurred, the LPV control is used to reallocate the control inputs to stabilize the vehicle. During normal work conditions, the LPV control is able to select energy efficient control actions. LPV-based robust controller was designed to deal with modeling inaccuracies and uncertainties, and further track the desired external yaw rate [170]. Besides, the tire force constraints were also taken into account. A two-degree-of-freedom LPV controller was investigated to implement traction control in independent in-wheel motor electric vehicle [171]. The simulations proved that the proposed LPV control worked properly under limited wheel slip ratio. Reconfigurable control based on LPV methods was developed to achieve velocity and path following for FWIA electric vehicles [172]. The main novelty of reconfigurable control is the accurate torque control and estimation.

HEVs have occupied an important position in the vehicle market these years, and many control strategies comprising LPV control are proposed to deal with the key problems of powertrain in HEVs. The schematic diagram of a typical powertrain system in HEVs is depicted in Fig. 7. Firstly, energy management is one of the hottest topics in HEVs. For instance, Wang et al. put forward a discrete-time LPV controller based on LFT to minimize the energy consumption in a parallel HEV [173]. The LPV control scheduled the gain with varying parameters, and split the usage of engine and battery successfully. Nevertheless, a two-layer LPV control method was developed for energy management of an HEV [174]. The authors aimed at determining the optimal control actions under inputs and states constraints and time-varying parameters. Model predictive control with LPV model was designed in [175] to improve the fuel economy of HEVs. The vehicle speed was considered as the varying parameter in LPV model. The effectiveness of the developed controller was verified with a MATLAB/SIMULINK simulation platform. Torque ripple reduction is another control issue of HEVs. The authors in [176] presented an LPV control framework to reduce the torque ripple in HEVs with Diesel engine and IC engine, respectively. In these works, the rotation speed was selected as a time-varying parameter, and internal model principle concerning multi-sinusoidal persistent disturbances was employed to enhance the LPV control performance. Some researches about the control of series HEVs have been also carried out through LPV control framework. The authors in [177] studied the control of a Diesel auxiliary power unit (APU) in an HEV. The nonlinear system, including a Diesel engine, a synchronous generator, and a three-phase diode rectifier, was represented by a quasi-LPV model. Then, a robust LPV control framework was used to solve the control problem. A reduced-order robust LPV control was introduced in [178] for APU control in a series HEV. In that paper, the nonlinear system was formed by a simple LPV model with parametric uncertainties, while the reduced-order LPV controller was developed based on LMI-based optimization. Finally, the effectiveness of the proposed gain-scheduled controller was illustrated with simulation results. In [179], a two-degree-of-freedom LPV controller was put forward to implement the torque vectoring in an HEV. The designed controller associated to an anti-windup control scheme could guaranteed the system stability and trajectory tracking performance.

In summary, LPV techniques are mainly applied to the control issues of electric vehicles powertrain, and only few works related to estimation. A robust LPV observer was proposed to estimate the thermally derated torque for HEVs in [181]. As the temperature variations would deteriorate control effect, it's necessary to estimate the thermally derated torque performance. Finally, the shortened FUDS test cycle was conducted to validate the designed observer. On the basis of the work in [181], the authors continued to develop an LPV controller to manage the thermally derated torque in HEV [182]. Besides, feedback field oriented control was utilized for torque control as well. The federal urban driving test cycle was carried out, and the results demonstrated the superiority of proposed observer and controller.

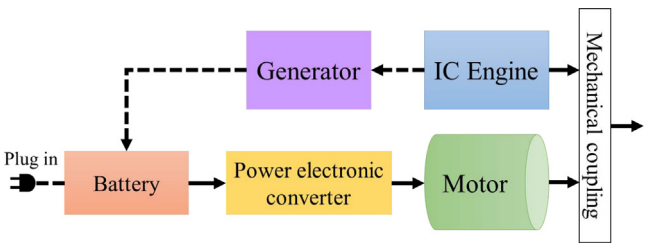


Fig. 7. Schematic diagram of a typical powertrain system in HEVs [180].

5.3. Aftertreatment systems

Diesel engine aftertreatment system is an important component for vehicle powertrain systems, which is usually used to eliminate the NO_x and the particulate matter (PM) in Diesel engine emissions. The aftertreatment system comprises of three parts: Diesel oxidation catalyst (DOC), Diesel particulate filter (DPF) and selective catalytic reduction (SCR) system. The schematic diagram of a typical aftertreatment system for current Diesel engines is shown in Fig. 8. The main function of DOC component is to convert CO to CO_2 and convert hydrocarbons to H_2O and CO_2. The DPF component is used to capture the particulate matters while the SCR system is employed to reduce the NO_x emissions.

Numerous model-based control strategies have been proposed for SCR systems to reduce the NO_x emissions and to constrain the ammonia slips, simultaneously. The principle of an SCR system is illustrated in Fig. 9. As reported in [183], considering the kinetic dynamics, the three-state SCR model can be established. Note that there are strong nonlinearities involved in this SCR model. Moan et al. proposed an adaptive LPV control strategy to simultaneously minimize the emission and the ammonia slip in urea-SCR systems [184]. In this work, a three-state nonlinear model with time-varying parameters was utilized. To improve the performance of the LPV controller, the authors proposed a quasi-LPV model to represent the complex nonlinear dynamics of urea-SCR systems. Moreover, the technique of principal component analysis (SPCA) was employed to reduce the complexity of the LPV model and the computational load for micro-control implementation. At last, performance comparisons between controllers based on a low-order model and a high-order model were conducted on GET-Power and MATLAB/SIMULINK platforms [185,186]. Temperature management through LPV control in SCR systems was put forward as well [187–189]. To this end, LPV state-space model was adopted to describe the temperature propagation in an SCR system, then a linear-quadratic-Gaussian (LQG) controller was developed for the temperature regulation. Furthermore, to achieve a better control performance, a Kalman filter-based observer was developed to estimate the system states. Finally, the LPV model and controller were validated under a simulator environment. A robust LPV controller was designed for the thermal management of a Diesel particulate filter [190]. For LPV control design, a simplified physical model was established. The performance of the designed LPV controller was compared with that of a baseline PID controller.

LPV-based observers have been also extensively applied for the estimation issues of aftertreatment systems. A Luenberger-like observer was developed to estimate the air fraction for Diesel engine and aftertreatment system [191]. The dynamic model for the Diesel engine and coupled aftertreatment were built in form of LPV model, which is efficient to represent the parametric uncertainties and disturbances. The designed observer was analyzed through Lyapunov functions and validated under real Diesel experiments. A nonlinear observer was promoted to estimate the immeasurable ammonia coverage ratio for an SCR system [192]. The nonlinear behaviors of the SCR system were first modeled in a quasi-LPV form. Then, an LPV observer was developed by stabilizing the estimation error system. Finally, an experimental test was con-

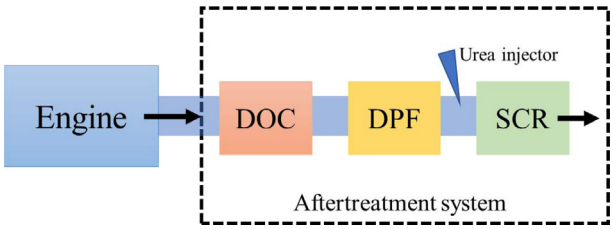


Fig. 8. Schematic diagram of a typical aftertreatment system for Diesel engines. DOC refers to Diesel oxidation catalyst, DPF is the abbreviation of Diesel particulate filter, SCR is selective catalytic reduction (SCR) system.

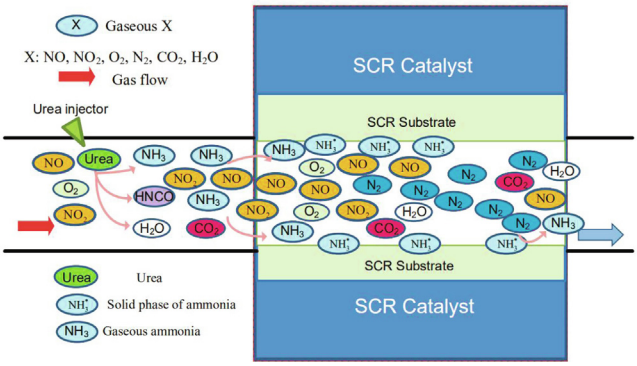


Fig. 9. Principle of a urea-based SCR system for Diesel engines [183].

ducted to validate the effectiveness of the designed LPV observer. Besides, an LPV proportional-multiple-integral (PMI) observer was investigated to estimate the NO_x sensor ammonia-cross-sensitivity factor, which is essential to compensate the sensor errors in SCR systems [193]. Applying Lyapunov stability theorem, the gain-scheduled PMI observer was developed while guaranteeing an $\mathcal{H}_\infty$ performance for the nonlinear error system. The simulation results proved that the LPV-based observer can provide a good estimation performance under disturbances. In addition, the authors in [195] studied a gain-scheduled Luenberger observer to estimate the NO and NO_2 concentrations in aftertreatment systems, including DOC and DPF, of Diesel engines. In this work, the nonlinear dynamics was modeled using LPV representations, and the observer was designed by analyzing the stability of the estimated error system. Similarly, ammonia and NO_x concentrations in a two-cell SCR system were also estimated through LPV-based techniques in [194]. Two observers were developed for the upstream cell of the SCR system, and one observer was designed for the downstream cell. The first observer in the upstream cell was used for unknown input and state estimation, while the second LPV observer was used to estimate the ammonia coverage ratio. The observer in the downstream cell aimed at estimating the NO_x emission and the ammonia coverage ratio. Finally, the excellent estimation performance of the designed observers was experimentally demonstrated.

6. Future research trends and challenges

Polytopic LPV techniques for IASs go toward maturity step by step, while some challenges still remain to be exploited. Moreover, with the emerging of new theories and hardware, more opportunities would follow constantly. Therefore, we focus hereafter on challenges and future trends related to polytopic LPV research topics from both theoretical and application viewpoints.

6.1. LPV complexity reduction

Polytopic LPV paradigms have become a standard formalism in stability analysis, estimation and control design of nonlinear systems [27,2]. From the theoretical viewpoint, it is possible to derive *necessary and sufficient* stability conditions for quasi-LPV systems [65]. However, in practice these stability conditions are conceptual rather than implementable since the computational burden swiftly increases in a way that most numerical solvers crash [66]. Hence, for stability analysis and control design of complex nonlinear systems such as IASs, it is crucial to study the reductions, either of LPV models or LMI constraints. At the same time, such numerical reductions must theoretically guarantee all specifications predefined for the initial model. A promising solution consists in exploiting data-based approaches such as SPCA-based technique [26] or deep neural networks based technique [196] to reduce the number of polytope vertices. As a result, the amount of LMI constraints, decision variables, online computational load and hardware resources requirements can be significantly reduced. The resulting integrated method of LPV model-based and data-based approaches would extend the application ranges of polytopic LPV techniques.

6.2. Fault detection and fault-tolerant control

With a constant increase in complexity, the demand on reliability and safety of IASs becomes more and more stringent. As a result, the issues of fault detection and fault-tolerant control have played a key role to minimize the performance degradation and to avoid dangerous situations [197]. Within this context, the ability of control reconfiguration is decisive to have a robust and resilient system operation, *i.e.*, actuator and/or sensor faults can be effectively dealt with while still guaranteeing an acceptable closed-loop performance [198]. A promising solution to this problem is based on a two-step design procedure for which a fault detection algorithm and a fault-tolerant control scheme are separately designed. To this end, polytopic LPV technique can be used to model virtual actuators and/or sensors for fault detection algorithm. Then, an LPV observer-based control scheme can be formulated with suitable closed-loop specifications to achieve the reconfiguration goal.

6.3. Limited capacities of perception and motion planning

Autonomous vehicles have been regarded as the future of vehicle industry. Quasi-LPV techniques have been successfully applied to path following control of autonomous vehicles [126,131,132,140], which is one of the most challenging problems in automated driving technology. For path following control, it is generally assumed that the perception is ideal and the vehicle can accurately obtain the positions of itself and of the surrounding obstacles. Then, the vehicle motion can be planned in function of the predicted obstacle trajectories. However, it always takes a certain time duration for the detection of the lane and the surrounding environment. Moreover, a smooth planned path requires an optimal iterative search which is also time-consuming [15]. Therefore, when an autonomous vehicle is with limited capacities of perception and motion planning, the planned path trajectory and the measured signals would be delayed. Finding an effective solution to obtain a robust path following control performance in presence of delayed signals would be the key to the widespread acceptance of autonomous vehicles.

6.4. Driver-automation shared driving control

Recent advances in actuation, perception technologies and artificial intelligence have prompted the intensive investigations driving assistance and highly automated driving in both academic and industry settings. However, fully automated driving is still prone to errors in the human presence in the control loop [199,200]. Driver-automation shared control has been shown as an effective scheme permitting to better meet the design guidelines of automation [201–204]. To this end, several novel shared control architectures, allowing for the driver-automation cooperation at the tactical level (decision-making level) and at the operational level (guidance level), have been proposed and experimentally validated under various driving scenarios [91,199]. The obtained results have shown a strong interest of the shared driving control concept in reducing both the driver workload and the driver-automation conflict. However, many challenges still remain which offer great opportunities for the research on shared driving control in the future, for instance.

- how to integrate the decision-making information into a robust vehicle control scheme to effectively handle hazardous situations (driver failure, undetected obstacles, sudden driving transition phases, etc.);
- how to integrate a *self-learning* ability into the shared control architecture such that the automation could *analyze* and *understand* the driver's actions during the shared control mode and the manual control mode.

Remark 8. Several survey articles related to this paper are available in the open literature, *e.g.*, the surveys on LPV theory [11,12], and on optimal control theory for vehicle dynamics applications [13]. Compared to the works in [11,12], which were done in 2000, we have introduced more recent techniques for the control design of LPV systems. In addition, the successful applications on IASs are special highlights. The survey in [13] mostly focused on linear control methods such as MPC control, LQR control for automotive systems. However, nonlinearities and uncertainties are unavoidable in IASs. In these cases, the surveyed approaches may not be suitable to deal with efficiently the related control problems. In our work, we have demonstrated the strengthes of polytopic LPV appraoches not only from the theory side, but also from the application side.

7. Concluding remarks

A review on recent advances in polytopic LPV approaches for IASs has been carried out. First, fundamental theories on polytopic LPV control were discussed. Some key techniques to derive less conservative results for stability analysis and robust control design were reviewed. Second, applications of polytopic LPV techniques to vehicle dynamics modeling, vehicle vertical dynamics control, vehicle lateral dynamics control, path-following control, and vehicle powertrain control were summarized. Finally, some challenges and future trends were given.

CRediT authorship contribution statement

Panshuo Li: Writing - original draft. **Anh-Tu Nguyen:** Writing - original draft. **Haiping Du:** Writing - review & editing. **Yan Wang:** Writing - review & editing. **Hui Zhang:** Writing - original draft.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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